Primary 6
Student Book



Math Term 1

2023 - 2024

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WORDS FROM THE MINISTER OF EDUCATION & TECHNICAL EDUCATION

Dear students and fellow teachers,

It gives me great pleasure to celebrate this crucial stage of comprehensive and sustainable development, an epoch in which all Egyptian people are taking part. This pivotal stage necessitates paving a foundation for a strong educational system which yields a generation that is not only capable of facing the major challenges the world is witnessing today, but one that also has complete possession of the skills of the future.

At a time when our world is witnessing successive industrial revolutions, the Egyptian state is keen on empowering its citizens by establishing a top-notch educational system that invests in its children the expertise required to get them to compete at both a regional and global level. This dictates that our educational system has at its core an emphasis on skills development, deep understanding, and knowledge production. This can only be done through modern curricula that keep up with the changes taking place globally— curricula which prioritize the development of skills and values, and the integration of knowledge. They are also curricula that focus on the provision of multiple learning sources, and integration of technology to enrich the educational process and to improve its outcomes, while addressing the most important contemporary issues.

To achieve this, we must all join hands to continue to revolutionize our education, and to support it with all that is required to transform it into a globally pioneering educational system.

My warmest regards to you, dear students, and my deepest gratitude to my fellow teachers.

Professor Reda Hegazy

Minister of Education & Technical Education

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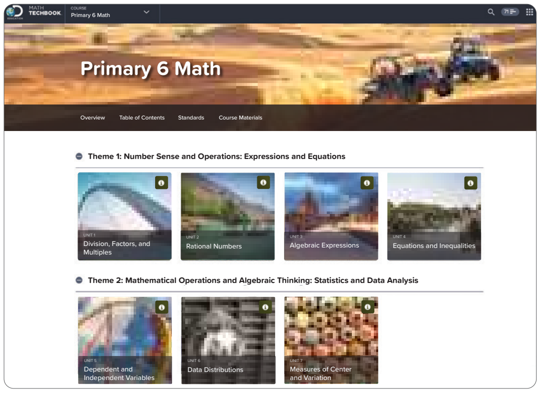
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Dear Parent/Guardian,



Welcome to Primary 6 Mathematics Techbook™! This comprehensive program inspires students to make sense of the world around them and to think and act like mathematicians. Throughout the digital and print program, students learn to reason mathematically, communicate using mathematical language, ask meaningful questions, solve complex problems, and work collaboratively with peers.

Primary 6 Mathematics Techbook was designed and written to teach to the Ministry of Education and Technical Education (MOETE) Primary 6 mathematics standards. The structure of Primary 6 Mathematics Techbook represents the Ministry's shifts in the Framework for Education 2.0, specifically focusing on accessing new and prior knowledge, building contextual understanding and procedural fluency, and making connections across mathematics to support application of skills and concepts. To help students make sense of mathematical content, the program also integrates a thematic approach and a variety of real-world scenarios.

Primary 6 Mathematics Techbook challenges students to build on what they learned in previous grades, applying concepts and skills in new ways. Students also learn new and complex concepts and skills that prepare them for the challenges of Preparatory 7 and beyond. Primary 6 students assume greater responsibility for their own learning and are encouraged to seek opportunities to apply the mathematics they are learning in the world around them.

The major work of Primary 6 includes ratios and proportional relationships, rational numbers (including operations with fractions and decimals), expressions and equations, geometry, statistics, and probability. Students also explore patterns, coordinate planes, statistical variability, and different ways to summarize and describe data distributions. Although these may seem like separate topics, students investigate and apply relationships among the topics to build a deeper understanding of each. They explore relationships between fractions and decimals, connect their understanding of two-dimensional figures to the Cartesian coordinate plane, and apply knowledge of mathematics to real-world situations through investigations. Students learned to think like mathematicians in previous years as they noticed patterns and rules and persevered to solve challenging problems. During this course, students are presented with lessons that elicit Classroom Practices that are aligned to thinking like a mathematician, such as representing and explaining their thinking, modeling their solutions, and striving for accuracy.

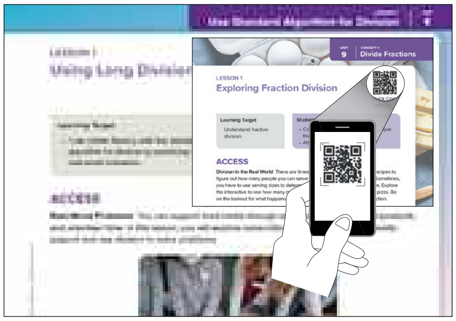
To inspire and motivate learning and curiosity, Primary 6 Mathematics Techbook features clear and engaging text, videos, digital tools, and Hands-On Activities. Hands-On Activities require students to investigate patterns and rules in mathematics and challenge them to communicate using mathematical language and models. The program also engages students in many kinds of writing and asks them to explain their reasoning and support their thinking using words, numbers, pictures, and symbols. When students engage in rich tasks that access prior knowledge and build reasoning, it is easier for them to make connections to the real world and to other mathematical learning.

Primary 6 Mathematics Techbook is divided into units. Each unit is divided into concepts, and each concept is divided into lessons. Each lesson has two main sections: ACCESS and BUILD AND CONNECT.

ACCESS Students activate their prior knowledge and begin to develop and express mathematical language.

BUILD AND CONNECT Students focus on communicating their understanding, reasoning, evidence, and mathematical strategies.
Students build deep conceptual understanding and a strong foundation for accessing knowledge in future lessons.

In addition, **WRAP-UP**, **PRACTICE**, and **CHECK YOUR UNDERSTANDING** features allow students to demonstrate their learning either verbally or in writing.



Within this Student Edition, you will find QR codes and Quick Codes that take you and your student to a corresponding section of Primary 6 Mathematics Techbook online.

We encourage you to support your student in using the print and online interactive materials on any device. Together, may you and your student enjoy a fantastic year of mathematics.

Sincerely,

The Math Team

UNIT
1

Theme 1 | Number Sense and Operations: Expressions and Equations

Division, Factors, and Multiples



Food Bank Mathematics

Unit Opener Video: Division, Factors, and Multiples

Quick Code
egm6001

Explains how multiplication and division can be used to solve real-world problems, using food banks as an example.

Key Vocabulary

Quick Code
egm6002

common denominator, distributive property, dividend, divisible, estimate, factor, greatest common factor (GCF), least common multiple (LCM), multiple, prime factor, prime factorization, quotient, relatively prime, Venn diagram

LESSON 1

Using Long Division in the Real World

Quick Code
egm6007

Learning Target

- I can obtain fluency with the standard algorithm for division by practicing with real-world scenarios.

Classroom Practices

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

ACCESS

Real-World Problems You can support food banks through donations of money, food products, and volunteer time. In this lesson, you will explore some information describing community support and use division to solve problems.



Food Bank

Let's Chat Discuss with a partner some ways you can apply division to supporting food banks.

BUILD AND CONNECT

Recognize Division Situations Which situations involve division? Identify all that apply.

A. The 78 volunteers at the food bank donated a total of 9,689 hours for the year. Each volunteer worked the same number of hours. How many hours did each volunteer donate to the food bank?

B. The food bank can make one food box that can feed 1 person 3 meals per day for 2 weeks. How many total meals can one food box make?

C. The food bank's top donor donated 1,250 tokens at each of 10 different fundraisers. How much money did the donor donate altogether?

D. With the 6,982 cans of food collected during the food bank's largest food drive, 93 meal boxes were created with the same number of cans in each box. In order for the food bank to use the most cans, how many would be in each box?

Let's Chat For each situation that you did not select, discuss with a partner the operation that should be used in the scenario.

Solve Division Problem I Solve the first division problem from the Recognize Division Situations activity, using the standard division algorithm. Show your full solution with all your work, including your initial estimate, and explain your answer. Be sure to include the meaning of any remainder that may result from the division.

Solve Division Problem II Solve the second division problem from Recognize Division Situations, using the standard division algorithm. Show your full solution with all your work, including your initial estimate, and explain your answer. Be sure to include the meaning of any remainder that may result from the division.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 2

Factorizing a number to its prime Factors

Quick Code
egm6015

Learning Target

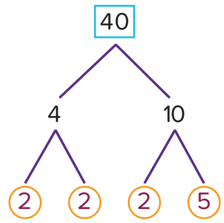
- I can utilize **prime factorization** in finding GCF and LCM.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

ACCESS

Prime Factorization You learned how to use factor trees to find the prime factors of numbers. Then, you wrote the prime factorization of the numbers. Are there other ways to find the prime factorization?

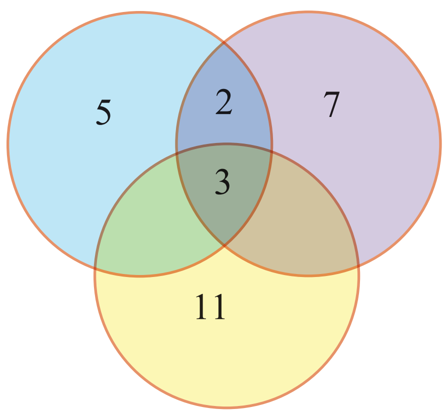


Let's Chat

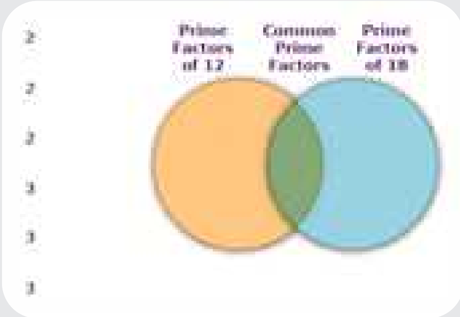
- What is the meaning of prime factorization?
- What are some methods that you can use to find prime factors?
- What are the advantages or disadvantages of these methods?

BUILD AND CONNECT

Venn Diagrams **Venn diagrams** can be used to compare similarities and differences between items. Where the circles overlap is where information is shared. Let's see how we can use prime factors to find GCF and LCM.



Whiteboard: Venn Diagrams Use the prime factorization of 12 and 18 to complete the Venn diagram. Demonstrate your understanding by drawing in your journal or using the digital tool.



12 and 18 Determine the prime factorizations for 12 and 18. If necessary, create a factor tree for each number.

12		×		×	
18		×		×	

Find the GCF and LCM Use your Venn diagram to answer these questions.

A. What is the GCF of 12 and 18?

B. What is the LCM of 12 and 18?

Let's Chat Examine the Venn diagram. Look at all the prime factors to see how they relate to the LCM and GCF. Discuss your findings with a partner.

Use Prime Factors Answer each question.

A. How can the Venn diagram help you find the GCF?

B. How can the Venn diagram help you find the LCM?

Relatively prime numbers are numbers whose only common factor is 1.

Let's Chat

- If you drew a Venn diagram for two relatively prime numbers, such as 4 and 9, what would it look like?
- If we made a Venn diagram to show the common prime factors of the two numbers, what factors would be in the overlap? Justify your answer.

Relatively Prime For two relatively prime numbers, which of the following statements are true? Identify all that apply.

A. Because there are no common factors in the intersection, the GCF is 0.

B. Because there are no common factors in the intersection, the GCF is 1.

C. The LCM is the product of the two numbers.

D. The LCM is the product of all prime factors in the Venn diagram.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Writing Expressions Using the G.C.F.

Quick Code
egm6017

Learning Targets

- I can write and analyze expressions involving the GCF.
- I can visualize how an expression that represents two whole numbers as a multiple of a sum of two whole numbers can represent a real-world situation.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.

ACCESS

Photo Credit: EugeniaSh / Shutterstock.com

Gathering Donations Charity Associations create food boxes to give to the community.



Gathering Donations

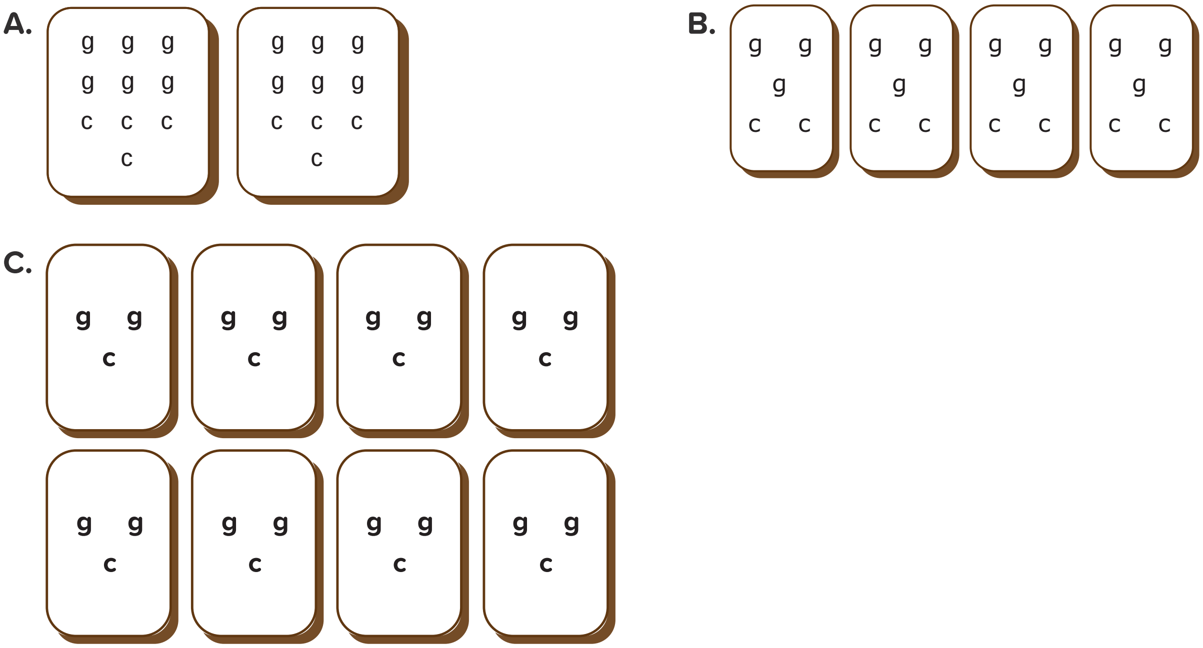
Let's Chat

- How can you use math to organize donations for food boxes?
- What kinds of problems would require using the GCF to solve?
- What kinds of problems would require using the LCM to solve?

BUILD AND CONNECT

Making Boxes A student has collected 12 grain bags and 8 packs of cheese in order to make donation boxes for the needy.

Organize the Boxes Help the student determine the greatest number of boxes she could make so that all boxes include the same number of items. Let g stand for a grain bag and c stand for a pack of cheese. You can represent this information with an expression.



Write the Expression Which expression represents the total number of items the student put into her boxes? Identify all that apply, and record the correct expressions.

A. $4 + 3 + 2$

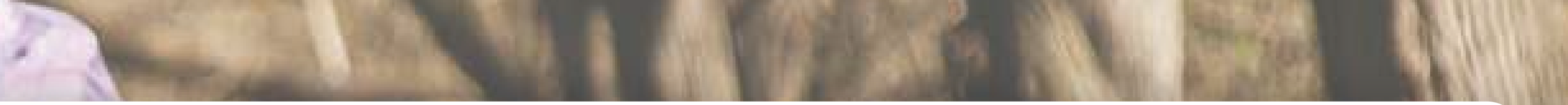
B. $4(3 + 2)$

C. $(4 \times 3) + (4 \times 2)$

D. $4 + (3 \times 2)$

Let's Chat Discuss with a partner your answer to the previous task.

- Did you choose more than one expression?
- If so, how are the expressions related?



Analyze an Expression Write your responses to the following questions.

A. Think about what $4(2+3)$ may stand for. Consider the number of food items you started with, 8 and 12, and the number of boxes you made, 4. How is 4 related to 8 and 12?

B. Consider the number of items in each box, 2 and 3. How do those numbers relate to the original problem?

The **distributive property** states that multiplying a number by the sum of two addends is the same as multiplying that number by each addend individually and then adding those products.

> **Let's Chat** Discuss with a partner what the distributive property means and how it relates to your responses in the previous questions.

Write Another Expression Is there another way to divide the items? Use a different number of boxes. Every box must have the same number of grain bags and the same number of packs of cheese. Write an expression to represent this information.

$$(______ + ______)$$

Making Baskets Students have collected 36 packs of cheese and 48 grain bags to make baskets. They will make as many baskets as possible with no items left over, and each basket will have the same number of packs of cheese and the same number of grain bags.

Basket Expression What might the expression $12(3+4)$ mean in terms of the baskets, packs of cheese, and grain bags?

Explain Why is the expression $12(3+4)$ helpful when representing this situation?

Analyze Another student is given 20 packs of cheese and 40 grain bags to make food boxes. He uses the expression $10(2+4)$ to represent how many boxes he could make with equal amounts of food in each box. His friend tells him that there is a way to make more boxes. Which one of these expressions would represent his friend's solution?

A. $20(2+4)$

B. $10(1+2)$

C. $10(1+4)$

D. $20(1+2)$

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 4

Analyzing Least Common Multiples

Quick Code
egm6019

Learning Targets

- I can analyze and evaluate addition and subtraction of fractions.
- I can use the least common multiple to create a **common denominator**.

Classroom Practices

- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Look for and make use of structure.

ACCESS

Fruit When you buy different types of fruit, the price may be the same, but the pieces inside the container may be quite different in size and number.



Fruit

Photo Credit: casanisa / Shutterstock.com

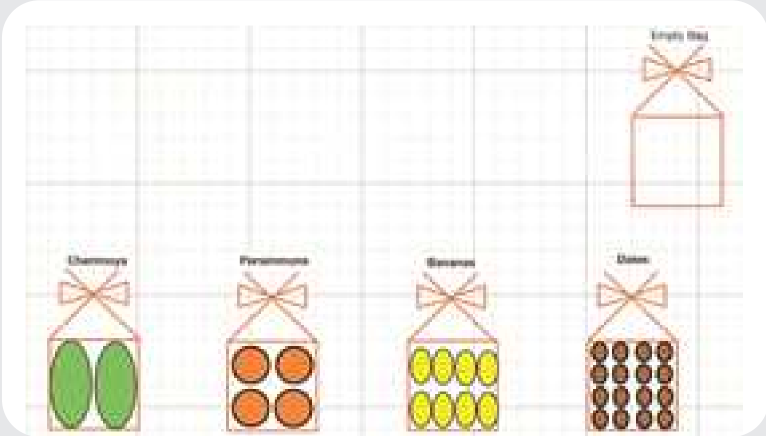
Let's Chat Share with your peers how you could evenly divide a pack of 32 grapes between 2, 4, and 8 friends.

BUILD AND CONNECT

Partitioning and the LCM A family purchased a set of plastic bags to store cut up fruit in for snacks and cooking. Each whole fruit has been cut into pieces to make a full package. The table shows a list of fruit and how many pieces make a full package. Think about how you would fairly share different fruit with a sibling if each of you have different packages of fruit.

Type of Fruit	Number of Pieces in a Bag
Cherimoya	2
Persimmons	4
Bananas	8
Dates	16

Whiteboard: Fruit Use the images of fruit in packages to complete questions. Demonstrate your understanding by drawing in your journal or using the digital tool.



Let's Chat Discuss with a partner how these fruits can be represented with fractions. Together, answer the following questions:

- In each fraction, what does the denominator represent?
- How can you write fractions to describe each package of fruit as it relates to the number of pieces that are in each fruit package?
- What fraction describes the number of pieces remaining if half of the package of dates is left?

Your Brother! has 5 packages of persimmons. He opened each package to give some from each to every family member, trying to find the ones that have the best taste..

The packages had the remaining portions left.

$$\frac{3}{4}, \frac{2}{4}, \frac{1}{4}, \frac{3}{4}, \frac{2}{4}$$

Use the Whiteboard: Fruit to help you work through the question. For each question, choose the correct amount from the given mixed numbers.

$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{3}{4}$

A. If you were to repackage the remaining pieces of fruit to create full packages of persimmons, how many packages of persimmons would remain?

B. Since there are 4 persimmon pieces in each of the 5 packages your brother opened, how many packages did he eat?

Your Sister and 3 of her friends have 4 packages of bananas . Each package contains 8 identical fruits, Each of them took a number of fruits to make banana pudding.

This is what remains of the banana packages.

$$\frac{3}{8}, \frac{2}{8}, \frac{5}{8}, \frac{7}{8}$$

Use the Whiteboard: Fruit to help you work through the question. For each question, choose the correct amount from the given mixed numbers.

$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$
$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$

A. If you were to regroup the bananas into packages, how many packages could be made from the remaining bananas?

B. How many packages were used?

Brother and Fruit Snack Your brother decided to try a new snack that required some persimmons and bananas.

This is what remained of the full packages once he finished. Use the Whiteboard: Fruit to help you work through the question.

Persimmons	Bananas
$\frac{3}{4}; \frac{2}{4}$	$\frac{3}{8}; \frac{2}{8}; \frac{5}{8}$

Your brother wants to collect the remaining fruit together. How many packages are left of each type?

Then find the sum of the two numbers together.

Your family had two packages of fruit, and the family ate some of each.

Now there are $\frac{3}{8}$ of a package of bananas and $\frac{1}{4}$ of a package of persimmons.

Use the Whiteboard: Fruit, markers, or paper models to make conjectures regarding each of the given questions. Be prepared to explain your reasoning.

A. How many packages are left for each type?

Let's Chat Discuss your reasoning and answers with a partner.

- How did your thinking change when the fruit were cut into different quantities per package?
- What would be the common size fruit amount you could use if you were determining how many complete fruit packages would be equivalent to the sum of these partial packages: $\frac{3}{4} + \frac{7}{12}$?
- How is the LCM related to the size of the fruit package you would use to make these fractions equivalent?

LCM Choose the LCM for each pair of numbers from the given values.

12	16	18	20	24	28	30
32	36	40	48	54	60	

A. _____ 4 and 12 **B.** _____ 5 and 8 **C.** _____ 6 and 10 **D.** _____ 6 and 9

Counting Fruit Evaluate these partial fruit combinations. The denominator describes the total number of pieces the package of fruit is divided into for pleasurable eating or easy sharing. You may also use Whiteboard, counters, or paper models to assist you in your reasoning, if needed.

As you reason these sums, think about the LCM of the denominators and how the LCM determines the "common fruit package size" that needs to be used to find the actual equivalent amount of a complete package.

Show your steps, and explain your reasoning.

A. $\frac{1}{4} + \frac{1}{12}$ **B.** $\frac{1}{5} + \frac{1}{8}$

Another Sum Recall the LCM of the denominator values. Create equivalent fractions using the LCM as the common denominator in each fraction, and find the sum.

Include your steps and explain your reasoning.

$$\frac{7}{10} + \frac{5}{6} =$$

Your Sum Creation Create your own fruit package fraction combination in the form of $\frac{a}{b} + \frac{c}{d}$, such that:

- The values *a, b, c,* and *d* all represent different whole numbers between 2 and 12, so each fraction value is less than $\frac{1}{2}$.
- The sum of your fractions is greater than or equal to $\frac{1}{2}$.

Then, enter the initial sum of two fractions as $\frac{a}{b} + \frac{c}{d}$.

Include your sum justification and final value using the LCM.

When directed by your teacher, trade your creation with a peer to find sums, comparing your reasoning and answers. Revise your answers or justifications as needed.

Please Share You buy one package of dates with 16 pieces. You have already eaten one piece when you remember that you owe your friend half of a fruit package.

$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$
$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$

Use the given amounts to answer these two questions.

A. What fraction represents the number of pieces you are to give to your friend.

B. After giving your friend his share, what fraction represents the amount of the fruit package that is left?

Now, choose from the given expressions and numbers to answer the next two questions.

$\frac{6}{16} - \frac{1}{16}$ $\frac{1}{2} - \frac{15}{16}$ $\frac{15}{16} - \frac{1}{2}$ $\frac{16}{16} - \frac{8}{16}$ $\frac{8}{16} - \frac{15}{16}$

2   4   6   8   12   16   24   36

C. What expression fits this scenario?

D. What is the LCM of the denominators in the expression?

Try This! Evaluate each expression using the LCM of the denominators. Justify your reasoning mathematically.

A. $\frac{5}{6} - \frac{3}{8}$

B. $1\frac{1}{12} - \frac{5}{9}$

Let's Chat When instructed by your teacher, discuss your reasoning of how you used the LCM for each expression with a partner. Revise and resubmit your answers, if needed.

Check Your Understanding Follow your teacher's instructions to complete this activity.


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UNIT

2

Theme 1 | Number Sense and Operations: Expressions and Equations

Rational Numbers



Rational Numbers

Unit Opener Video: Rational Numbers

Explores how rational numbers are utilized in daily life.

Quick Code egm6021

Key Vocabulary

absolute value, fraction, number line, opposite (number), rational number, set

Quick Code egm6022

LESSON 1

Using a Number Line to Describe Data

Quick Code
egm6023

Learning Targets

- I can recognize the **number line** to include negative numbers in order to utilize them to model real-world situations.
- I can plot points representing positive and negative numbers.

Classroom Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

ACCESS

Grocery Shoppers A **rational number** $4, 3\frac{1}{2}, \frac{1}{2}, 0,$ 2.25 can be made by dividing two integers. When you shop for groceries, there are rational numbers everywhere. Think about the quantity of an item or a price.


Packs of Biscuits

Photo Credit: mrinalpal / Shutterstock.com

Let's Chat Discuss with your classmates the ways in which rational numbers are a part of your life.

- How do you use rational numbers when you are shopping?
- Do you use them to describe any activities?

As you think about how rational numbers are a part of your everyday life, let's first learn how using a number line can help us recognize rational numbers (integers - fractions - decimals -mixed numbers - ...).

BUILD AND CONNECT

Using a Number Line In Egypt, the height of the land ranges from 145 meters below the level of the sea in the Western Desert to 2,875 meters above the level of the sea in the Sinai Peninsula. If you are traveling across Egypt from the Sinai Peninsula to the Western Desert, you will start at an elevation of 23 meters and will decrease by 25 meters by the end of the week.

Positional Distance Sketch a blank, unlabeled number line to graph each location and answer the following questions.

A. How did you graph the position of the Sinai Peninsula?

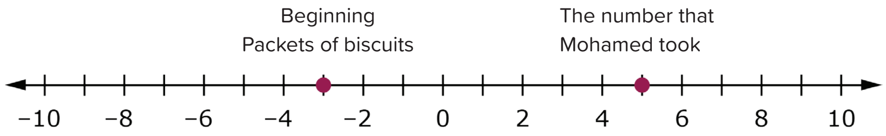
B. How did you graph the Western Desert's location?

Analyze Another Problem Mohamed and Hoda’s father purchase 10 packs of biscuits every week to be shared equally between Mohamed and Hoda. While Mohamed asked his sister to get some packs of biscuits one week and he will return it back to her the following week.

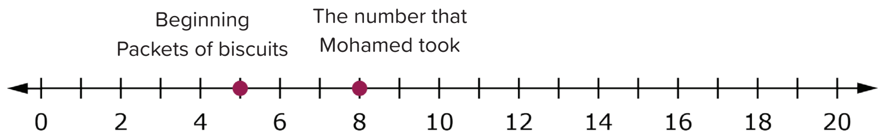
If Mohamed actually ate 8 packs of biscuits this week.

Which number line correctly shows the number of packets Mohamed ate this week.

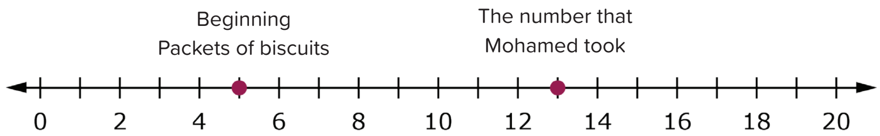
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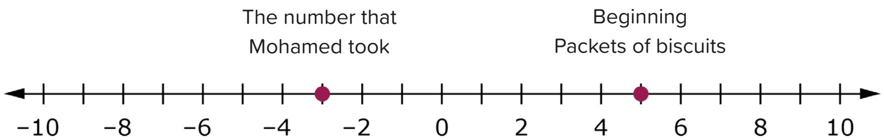
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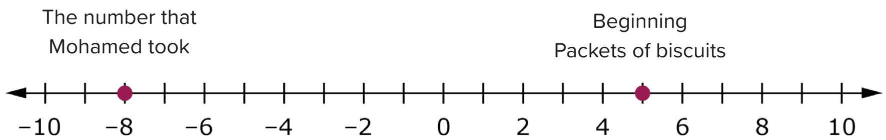
3.



4.



5.



Let’s Chat Think about the number lines and discuss the following questions with a partner.

- What did 0 indicate on each number line?
- What do you notice about the numbers to the left of 0? What do those values mean in the context of the problem?

Plotting and Comparing Numbers On a cold day, a fisherman is out fishing.



Fishing on a Cold Day

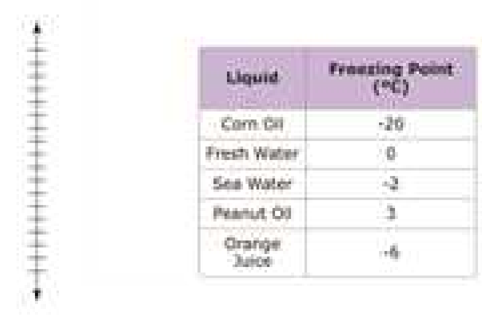
At what temperature does sea water freeze?

Fresh water freezes at 0 degrees Celsius, but other liquids, such as sea water, freeze at other temperatures. The table shows the temperature at which some liquids freeze.

Liquid	Freezing Point (°C)
Corn Oil	–20
Fresh Water	0
Seawater	–2
Peanut Oil	3
Orange Juice	–6

You can graph the temperatures using a horizontal or a vertical number line. Use one of the following Whiteboards with your class to view and discuss the two graphing options. Then, choose the orientation of the number line that you want to use to graph the given freezing points.

Whiteboard: Vertical Number Line Choose a scale, label the number line, and graph the given freezing points on the vertical number line. Demonstrate your understanding by drawing in your journal or using the digital tool.



Liquid	Freezing Point (°C)
Corn Oil	-20
Fresh Water	0
Sea Water	-2
Peanut Oil	3
Orange Juice	-6

Whiteboard: Horizontal Number Line Choose a scale, label the number line, and graph the given freezing points on the horizontal number line. Demonstrate your understanding by drawing in your journal or using the digital tool.



Liquid	Freezing Point (°C)
Corn Oil	-20
Fresh Water	0
Sea Water	-2
Peanut Oil	3
Orange Juice	-6

Graph the Temperatures Draw the number line you chose. Then, write your responses to the following.

A. Explain why you chose the number line you did and the scale you used.

B. Which liquid had the highest freezing point? Which had the lowest?

Order the Liquids Make a chart similar to the one shown and order the liquids by their freezing points, from the liquid with the lowest freezing point to the liquid with the highest freezing point.

peanut oil	sea water	fresh water
corn oil	orange juice	

Lowest				Highest

LESSON 2

Using a Number Line and Symbols to Compare Numbers

Quick Code
egm6025

Learning Targets

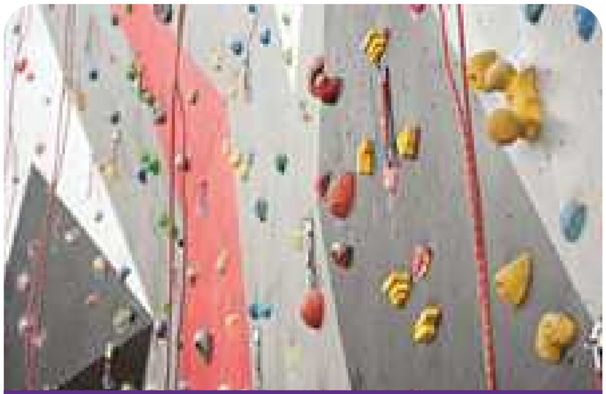
- I can discuss the relative positions by plotting points that represent positive and negative numbers on a number line.
- I can utilize the interactive to discover **opposites** of numbers.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.

ACCESS

Rock Climbing Suppose a group of people go indoor rock climbing together. There is an elevated platform in the middle of the giant rock wall. Each climber's location is represented by their distance from the platform. The guide keeps track of each person's position. The following table shows the positions of each of the climbers relative to the platform.

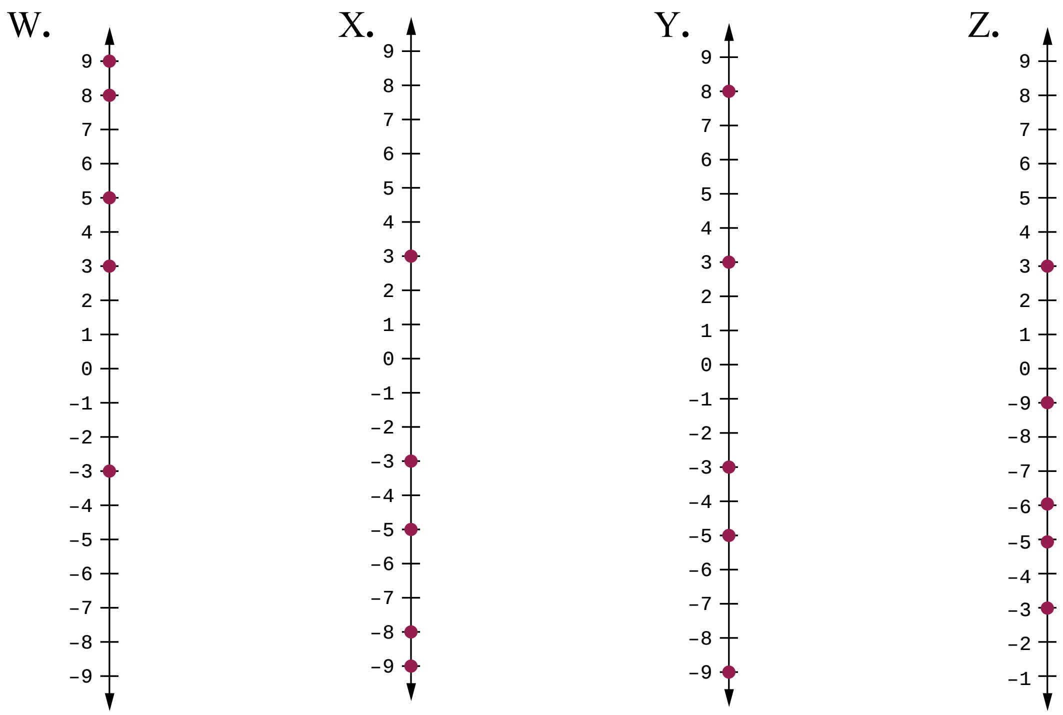

Rock Wall

Person	Position from Platform (meters)
Climber A	–5
Climber B	3
Climber C	–9
Climber D	–8
Climber E	–3

BUILD AND CONNECT

Plot and Compare Numbers Before you get started, think about where the platform is on the number line. What do negative numbers represent in the scenario? How about positive numbers?

Graph Positions Choose the number line that correctly shows each person's location.



Interpret Locations Use the table that displays each rock climber's position to complete the following statements.

The person who is below the platform but closest to the platform is Climber _________.

The person who is the farthest away from the platform is Climber _________ because _________ is more units away from 0 than all of the other climber's locations.

Climber _________ and Climber _________ are the same distance away from the platform because their locations are the same number of units away from 0 on the number line.

Now that you have ordered and analyzed the climbers' positions, think about where you would rather be if you were a climber: at –12 or –8? Why?

If you were at –3 and knew another climber was the same distance away from the platform, but not at your location, where must that other climber be?

Use Symbols to Compare Numbers You have used number lines to order and compare numbers. Now, use what you know to compare numbers using symbols.

Compare Numbers Write each statement, filling in each blank with the inequality symbol, < or >, that correctly completes the statement.

A. –7 ________ –3

B. 8 ________ 0

C. 3 ________ 4

D. 2 ________ –9

E. –6 ________ 0

F. 2 ________ –2

Let's Chat With a partner, use a number line to help explain your reasoning about the symbol you chose for each statement.

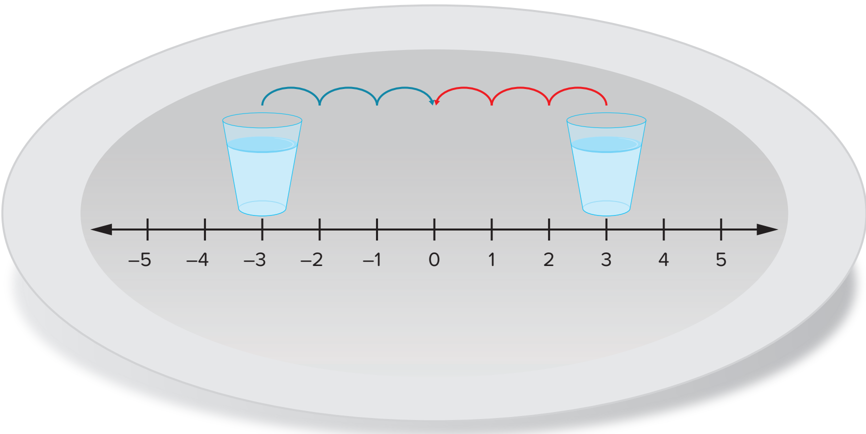
Can You Have the Opposite of a Number (Additive Inverse)? Help the waiter balance his tray while investigating opposites of numbers.

Quick Code egm6025

Interactive

Tray Matcher Interactive

Go online to explore this interactive activity.



Definition Use what you learned to write a definition of *opposite numbers*.

Use mathematical language and include an example with your definition.

Opposites Record each given number and then write its opposite.

A. –5, ________

B. 6, ________

C. $\frac{1}{2}$, ________

D. –16, ________

E. $-\frac{3}{2}$, ________

Balancing Why do you think opposite numbers balance? Explain your reasoning.

Which Are True? Select all the true statements.

- **A.** A number and its additive inverse are the same distance away from zero on a number line but on opposite sides.
- **B.** All opposites are negative numbers.
- **C.** Zero is its own additive inverse.
- **D.** To show 5 and its additive inverse on a number line, count 5 units and plot the point 5 units to the right of 0. Then, plot the point 5 units to the left of 0.
- **E.** The additive inverse of any number is zero.

Let's Chat Discuss with a partner how you would revise each false statement to make it true.

Additive Inverse of a number A student said that she thinks of a negative sign as indicating the additive inverse of a number. She says that two negative signs, such as $-(-3)$, would therefore mean "the additive inverse of 3" or "the additive inverse of −3." Use her idea to explain how to find the opposite of an opposite.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Analyzing Rational Numbers by Using Models

Quick Code
egm6029

Learning Targets

- I can utilize the visual of a Venn diagram to help students conceptualize the number system.
- I can investigate the symmetry of the number line and the use of opposites through a real-world context of tug-of-war.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

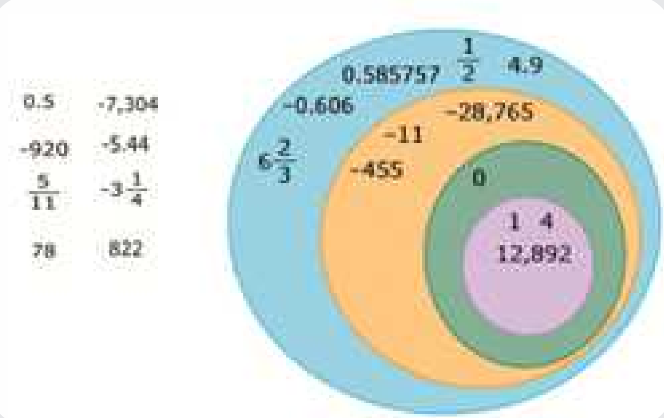
ACCESS

Numbers in Daily Life Have you ever looked at a thermometer If so, you may have seen numbers on the thermometer that help us know the temperature. You will even see negative numbers on a thermometer. If the temperature falls between two lines then it could be also represented as a fraction or a decimal. You experience different types of numbers in your everyday life all the time.



Thermometer

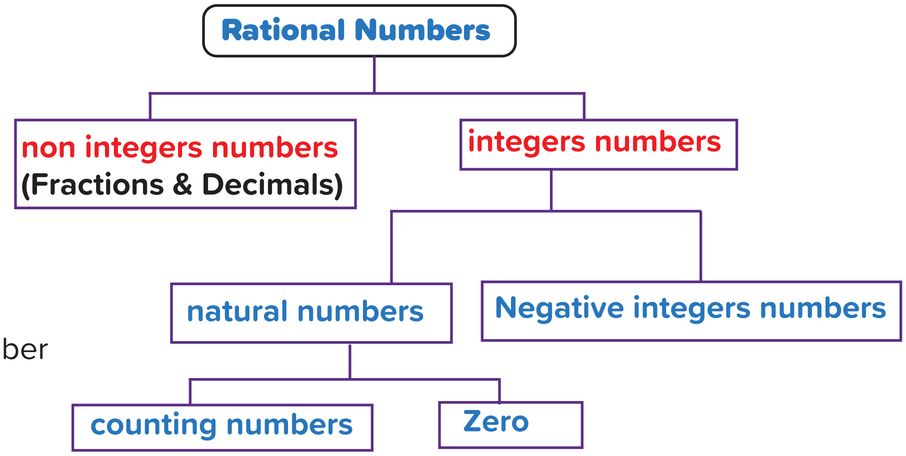
Whiteboard: Venn Diagram "Place the numbers into the subset where you think they fit best." Demonstrate your understanding by drawing in your journal or using the digital tool.



Let's Chat Discuss what you think the diagram means with a partner.

BUILD AND CONNECT

Sets of Numbers When placing numbers into groupings for rational, integer, natural, or counting numbers, it is important to place the number in its best set. Subsets are within sets. Example: Counting numbers includes the number 2 even though it is also a natural, integer and rational number. Being that counting numbers' set is within the natural set and natural number set is within the integer set and the integer set is within the rational set.



Relating Rational Numbers

Placing Numbers Describe the criteria that you used to place each number in Whiteboard: Venn Diagram.

rational numbers	integers
natural numbers	counting numbers

Naming Numbers Match the numbers to the best subset.

A. 0.585757, ________

B. 4, ________

C. –455, ________

D. 0, ________

E. –0.606, ________

F. $6\frac{2}{3}$, ________

G. –11, ________

H. –28,765, ________

I. $\frac{1}{2}$, ________

J. 1, ________

K. 12,892, ________

L. 4.9, ________

Let's Chat Share your answers to the previous problem with a partner, revising as needed. Then, discuss the sets of numbers you just named and the criteria that you used to name each set. Discuss the following questions.

- In which subsets is the number 14? How about –3 and 0.5?
- How are the different sets of numbers related to each other?
- What means by belonging an element to a set?
- What means by subsets and including a set to another set?

Analyze Relationships Identify the statements that are always true. Select all statements that apply.

A. All integers (Positive , Negative and Zero) are also natural numbers.

B. All counting numbers are also natural numbers, integers, (Positive , Negative and Zero), and rational numbers.

C. All rational numbers are also integers (Positive , Negative and Zero).

D. All natural numbers are also integers (Positive , Negative and Zero) and rational numbers.

E. All integers (Positive , Negative and Zero) are also rational numbers.

F. All rational numbers are also natural numbers and counting numbers.

Let's Chat

- Based on the work you've done so far, what do you think would be the definition of a *rational number*?
- Compare your definition to the glossary definition.

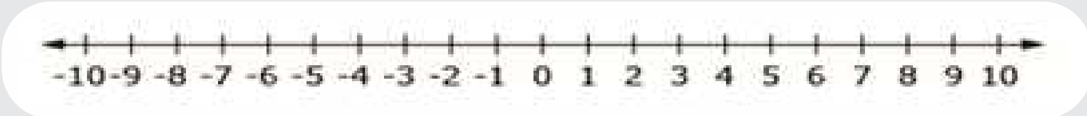
Writing Rational Numbers Write the given rational numbers in fraction form, $\frac{a}{b}$.

A. 4, ________ **C.** –45, ________

B. 0.75, ________ **D.** –1.5, ________

Use what you know about rational numbers to investigate plotting these values on a number line.

Whiteboard: Number Line Demonstrate your understanding by drawing in your journal or using the digital tool. Use this Whiteboard to help you complete the exercises. First, you will plot the given numbers on the number line. Then, you will plot the opposite of each number.



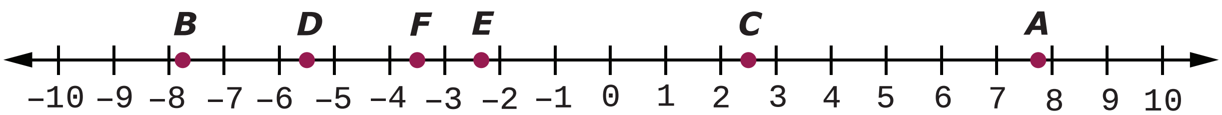
Plot Rational Numbers on a Number Line Draw the number line shown and plot the numbers 1, 2.5, $\frac{1}{2}$, 5.5, and $6\frac{3}{4}$. Then, explain your reasoning for each of the following questions.

A. Describe how you identified the location of each number.

B. Use the number line you just created to plot the opposite of each number. Describe how you plotted the opposites.

Let's Chat How did plotting the positive points first help you to find their opposites?

Error Analysis A student tried to plot the following six numbers on a number line. Which points did the student plot correctly? Record all that apply.



A. $7\frac{3}{4}$ **D.** –5.5

B. –8.25 **E.** $-2\frac{1}{3}$

C. 2.5 **F.** $-4\frac{1}{2}$

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 4

Comparing and Ordering Rational Numbers

Quick Code
egm6031

Learning Target

I can practice representing real-world situations as rational numbers, and then order the values from least to greatest.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

ACCESS

Ordering Numbers Think about how you can apply what you know about rational numbers to compare and order their values.



Rational Numbers

Photo Credit: matimix / Shutterstock.com

How can you compare and order rational numbers?

Let's Chat Discuss rational numbers that you encounter in the real world.

- Where do you encounter rational numbers?
- Are the numbers integers, fractions, or decimals?
- Name several examples of rational numbers.
- What strategies could you use to compare and order these numbers?

BUILD AND CONNECT

Compare and Order Rational Numbers

Comparing Rational Numbers In the Hands-On Activity, you will interpret situations using rational numbers, and then order those numbers from least to greatest. Use this online digital resource for Hands-On exploration.

Let's Chat When you have completed the activity, think about how you and your group were able to order the numbers from least to greatest.

- What process did you use?
- Do you think all of the groups in your class used the same process?

Order Rational Numbers Order the given set of numbers from least to greatest, using a table like the one shown.

2.1   1.4 $-3\frac{1}{4}$ $-1\frac{7}{8}$ $-2\frac{1}{2}$

Least				Greatest

Explain Your Steps Describe the steps you would take to order a set of rational numbers.

Think about all the numbers that could be on a number line between the tick marks.

What Numbers Would Fit? For each pair of numbers, draw a number line with appropriate labels. Then, write a number that is located between each pair of numbers. Justify your reasoning using your number lines.

A. 3.75 and 3.76

B. –9.1 and –9

C. $-\frac{3}{4}$ and $-\frac{1}{2}$

D. $\frac{1}{9}$ and $\frac{2}{9}$

Let's Chat Discuss with your partner

Ordering numbers on the number line as the following.

- Arranged ascendingly (from the smallest to the greatest), If we move from left to right.
- Arranged descendingly (from the greatest to the smallest), If we move from right to left.
- To compare between any two numbers, the number which is located to the right of the other is the greater?

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 5
Exploring Absolute Value

Quick Code
egm6035

Learning Targets

- I can represent distance for real-world situations involving jumping fish and meeting friends at a fishpond.
- I can understand **absolute value** on a number line.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.

ACCESS

Field Trip Your class is on a field trip at a nature center. The tour guide is showing you a type of fish that can dive below the surface of the water and jump above the surface of the water. He keeps track of the dives and jumps in the following table.


Fish

Photo Credit: Al McGlashan / Shutterstock.com

Observation	Dive (In Meters)	Water Level (In Meters)	Jump (In Meters)
A	3	0	3
B	2	0	2
C	4	0	4

Your friend thinks the guide should have used negative integers to represent the dives. Your friend asks the guide why he used positive integers to represent the dives and why the water level is 0.

BUILD AND CONNECT

Part A: Analyze Data on a Number Line

Use Number Lines to Show the Data Show the fish data as integers on a number line. Label each point with the observation letter. Then, explain what you notice about the points.

Analyze Another Situation Suppose your friend wants you to meet her at the fishpond at 2:30 p.m. You arrive at 2:25 p.m. Your friend arrives at 2:35 p.m.

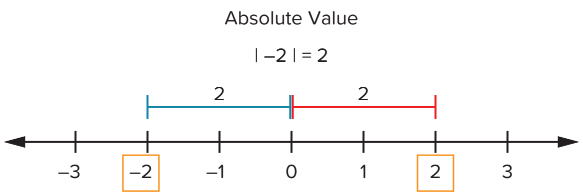
Represent this scenario on a number line and explain your reasoning. Instead of using the given times, represent the elapsed time and the meeting time as integers.

Select the True Statements Assuming that zero represents the arrival time, select all of the statements that are true.

A. Your arrival can be represented as –5 on the number line.
B. Your arrival can be represented as 5 on the number line.
C. Your arrival can be represented as a point that is 5 units from 0.
D. Your friend's arrival can be represented as –5 on the number line.
E. Your friend's arrival can be represented as 5 on the number line.
F. Your friend's arrival can be represented as a point that is 5 units from 0.

Let's Chat Discuss your thoughts with a partner.

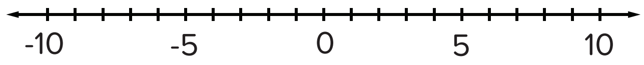
Part B: The absolute value is the distance between the position of any number and the position of zero on the number line, and it is always positive or equal to zero.



Saleh, Omar, Nour and Tarek are brothers who went on an outing with their father to an amusement park. Their father bought them the same number of tokens, but some of them played more than others and had to borrow tokens from their brothers to continue playing games. The table represents how many tokens each brother had at the end of the day.

Name	Ending Token Count
Saleh	10
Omar	–8
Nour	–2
Tarek	8

Whiteboard: Game Tokens Use the table to label the number line with the ending number of tokens for each person. Demonstrate your understanding by drawing in your journal or using the digital tool.



Let's Chat

- Talk with a partner about absolute value.
- What did you notice about the distance each person was from 0?

Game Tokens Using the information gathered in the Whiteboard, respond to these questions.

A. Which person was the farthest away from 0?

B. Where there any people equally distanced from 0? If so, how far were they from 0?

Definition Write a conjecture for each of the following.

A. What do you think is the definition of *absolute value*?

B. What do you think this statement means? $|-3| = 3 = |3|$

Absolute Strategy Record all of the true statements.

A. If numbers are opposites on the number line, then they have the same absolute values.

B. The highest possible absolute value is further away from zero.

C. The smaller the absolute value, the closer the number is to zero.

D. The larger the absolute value, the closer the number is to zero.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 6

Comparing Absolute Values

Quick Code
egm6037

Learning Targets

- I can compare absolute values using symbols.
- I can interpret the use of absolute value through real-world situations involving money and temperature.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

ACCESS

Absolute Values Think about some values you have compared, such as $-\frac{5}{6}$ and $\frac{2}{3}$ .Now, think about comparing the absolute values of these rational numbers. Do you think the comparisons will be the same or different?

Absolute Value

$\left|-\frac{5}{6}\right| = \frac{5}{6}$

$\left|\frac{2}{3}\right| = \frac{2}{3}$

BUILD AND CONNECT

Part A: Compare Values

Compare Absolute Values Write each statement using <, >, or =.

A. $|-4|$ ________ $|-3|$

B. $|-8.2|$ ________ -7.9

C. -1.4 ________ $|-1.4|$

D. $5\frac{5}{6}$ ________ $\left|-\frac{35}{6}\right|$

E. $\left|9\frac{3}{5}\right|$ ________ $\left|-9\frac{3}{4}\right|$

F. $|-2.71|$ ________ 2.7

Significant Symbols Use this absolute value statement to help you choose the term that correctly completes the sentences.

$$|2| = |-2|$$

absolute value	distance	direction
equals	negative	positive

A. The ________ symbol in the expression $|2|$, signifies the distance from 0 to 2.

B. The ________ symbol in the expression $|-2|$, signifies the distance from 0 to -2.

C. The ________ sign shows the relationship between each side and that their values are the same ________ from 0.

Each symbol in a statement has importance and contributes to the meaning of the mathematical sentence.

Justify Mouataz told his friend that absolute values are never negative. Is he correct? Explain why or why not.

Conclusion Fill in the blanks to make the statements true.

A. A negative number with an absolute value greater than 16 is ________.

B. Absolute values of opposites are ________.

C. The farther a number is from zero, the ________ the absolute value.

D. The smaller the absolute value, the ________ the number is to zero.

Part B: Comparing Real-World Values

On a hot day, suppose you borrowed 6.50 LE from your friend Naglaa to buy a big bottle of water. Later on, you borrowed another 3.25 LE from your friend Rawan to buy a small bottle of water.

Think about how you could use absolute value to represent and compare the amounts of money you borrowed.



Water Bottles

Photo Credit: Pressmaster / Shutterstock.com

Compare Values Suppose Rawan says that she can represent the amounts in this situation with the numbers 6.50 and –3.25. She says that because $-3.25 > 6.50$, you owe her more money than you owe Naglaa.

Is Rawan correct? Write your conclusion and explain your reasoning.

Compare More Values In a lab, there are two freezers set at different temperatures to preserve specimen. Freezer A is –17°C and Freezer B is –33°C. Answer each of these questions.

A. Which number is greater?

B. Which temperature is colder? Explain how you know.

Let's Chat Share your answers with a partner, revising them as needed. Then, discuss these questions.

- What are the differences between the two questions in Compare More Values?
- How might you determine what kind of comparison is appropriate for a given situation?

Represent Statements Answer each question by choosing one of the given number comparisons that correctly represents the answer.

$-22 < -5$	$-22 > -5$	$-22 < -5$	$-16 < -6$
$-16 > -6$	$-16 > -6$	$-4.8 > -4.88$	$-4.8 > 4.88$

A. It is -5°C in Freezer A and -22°C in Freezer B. Which freezer has the lower temperature?

B. Lake A has a positional distance of -16 meters, and Lake B has a positional distance of -6 m. Which lake is located farther below sea level?

C. Two rational numbers are -4.88 and -4.8. Which number is greater?

Order Positional Distances The positional distance of several ponds are given. Create a table like the one shown to order the positional distances from closest to sea level to farthest from sea level.

Pond	Positional Distance (In Meters)
A	-28
B	-430
C	33
D	89
E	-214

Closest to Sea Level				Farthest from Sea Level

Let's Chat Share your answers with a partner and explain your reasoning.

Check Your Understanding Follow your teacher's instructions to complete this activity.

UNIT 3

Theme 1 | Number Sense and Operations: Expressions and Equations

Algebraic Expressions



Algebraic Expressions

Unit Opener Video: Algebraic Expressions

Quick Code egm6039

Explores several real-world situations that can be described using algebraic expression.

Key Vocabulary

algebraic expression, Associative Property, coefficient, Commutative Property, constant, Distributive Property, equivalent expressions, evaluate, expressions, like terms, order of operations, variable

Photo Credit: (a) Oleksiy Mark / Shutterstock.com; (b) Sari Sanee / Shutterstock.com

LESSON 1

Creating Mathematical Expressions

Quick Code
egm6041

Learning Targets

- I can use a **variable** in a mathematical **expression** to communicate multiple pieces of data.
- I can define the elements of **algebraic expressions** such as term, **like terms**, **constants**, and **coefficients**.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

ACCESS

Life in Space When visitors enter the planetarium, they are surrounded by images and sounds that make them feel like they are astronauts floating in space.



Planetarium

Photo Credit: Pavel Gabzdyl / Shutterstock.com

Let's Chat How would daily life in a spaceship be different than life on Earth?

BUILD AND CONNECT

Part A: Expressions and Variables

An Astronaut's Height In addition to the changes in daily life that an astronaut encounters in space, gravity changes an astronaut's height.

Effects of Gravity Astronauts are approximately 0.05 meters taller while traveling in space than they are on Earth. Complete the table to determine the height of some astronauts while traveling in space.

Height on Earth (meters)	Height in Space (meters)
1.65	(A) ______
1.73	(B) ______
1.80	(C) ______
1.84	(D) ______

Let's Chat How did you find the heights of the astronauts while they were traveling in space? Explain how you can write an expression to represent each astronaut's height.

Analyze the Situation Think about the relationship between the height of an astronaut on Earth and the astronaut's height in space. Respond to these questions.

A. Think about what values are always the same in this situation and what values vary. What values vary depending on the astronaut?

B. Which value in this situation stays the same each time you find an astronaut's height in space? Explain your reasoning.

Build an Expression Suppose the height of an astronaut while traveling in space is measured and you want to find the astronaut's height on Earth. Use the given terms and operations to create a mathematical expression that represents the height on Earth if the height in space is *h* meters.

h	$-$	$+$	0.05	1.73

Let's Chat The letter *h* in the expression you just created is called a variable. Why do you think it is called a variable? What do you think a variable is?

Analyze Other Situations An astronaut's weight and the amount of time the astronaut sleeps is also affected when the astronaut goes on space missions.

Weight on the Moon Some astronauts have been sent on missions to the moon. An astronaut on the moon weighs $\frac{1}{6}$ of what the astronaut weighs on Earth.



Astronaut

Photo Credit: Oleg_Yakovlev / Shutterstock.com

Show your calculations for the weights of some astronauts while visiting the moon.

Weight on Earth (Newton)	Weight on the Moon (Newton)
66	(A) ______
84	(B) ______

Write an Expression Think about the relationship between the weight of an astronaut on Earth and his weight on the moon, as shown in the table you just completed. Use the information in the table to complete these three tasks.

A. Which value varies depending on the astronaut?

B. What part of the relationship between the weight of an astronaut on Earth remains the same each time you find the weight of an astronaut on the moon?

C. Write an algebraic expression that would represent the weight of any astronaut while visiting the moon. Select a letter that you can use to represent the weight of an astronaut on the moon using the astronaut's weight on Earth.

Sleeping in Space Astronauts must strap themselves in when sleeping so that they don't float around. They schedule 8 hours of sleep each day they are in space.

Write an expression that represents the number of hours an astronaut sleeps during any mission, where the number of days is the unknown number. Explain your reasoning.

Part B: Sort Expressions

Sort Expressions Sort the given expressions into two different groups. Clearly explain what criteria you used for the two groups.

9	$2+7.8$	$3(6)+2$	$7(1.4+3.2)$
$48-1$	$\frac{1}{4}m-2$	$2n$	$3p+4q$
$5x+3x-1$	$x-36$	$r+s-t$	

A. What is your first group of terms?

B. What is your second group of terms?

Analyze Suppose a student sorted the expressions into two groups, as shown in the table.

Group 1	Group 2
$\frac{1}{4}m-2$	9
$2n$	$2+7.8$
$3p+4q$	$3(6)+2$
$5x+3x-1$	$7(1.4+3.2)$
$x-36$	$48-1$
$r+s-t$	

Explain your reasoning for these questions.

A. What criteria do you think was used to sort the expressions this way?

B. The first group contains only algebraic expressions. The second group contains only numeric expressions. How would you define an algebraic expression? A numeric expression?

Let's Chat Share with a partner your answers in Sort Expressions and Analyze. Revise your answers as needed. Be prepared to share your findings with your peers.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 2

Analyzing Mathematical Expressions

Quick Code
egm6043

Learning Target

- I can define the elements of algebraic expressions such as term, like terms, constants, and coefficients.

Classroom Practices

- Look for and make use of structure.

ACCESS

Language of the Universe It has been said that math is the “language of the universe.”



Speaking Math

Photo Credit: Ebtikar / Shutterstock.com

Let’s Chat

- In what way is that true?
- How does math describe the universe?
- How do you become fluent in this language?

BUILD AND CONNECT

The Language of Expressions Let’s start by exploring the similarities and differences between expressions and equations.

Let's Chat Discuss the following with a partner:

- What is the difference between an expression and an equation?
- Describe a real-life situation that can be modeled by an expression.
- How would you change the situation so that it could be represented by an equation?

Understand Terms Every expression that you encounter has terms, and some have like terms. Look at the first five examples in this table to see if you can determine what terms and like terms are. Then, identify the number of terms and like terms for the remaining expressions.

Expression	Number of Terms	Like Terms
5	1	None
$8 + 2$	2	8, 2
$x + 12$	2	None
$4n + 2n + 2$	3	$4n$, $2n$
$6 + 3x + 3$	3	6, 3
$m + 3 + 2n + 2$	(A) ______	(B) ______
$16x + 2x$	(C) ______	(D) ______
$8z + 3z + 9$	(E) ______	(F) ______
$7x + 7x + 1 + 2x$	(G) ______	(H) ______

Let's Chat Discuss your answers to the last exercise with a partner. Then, discuss how you would define *terms* and *like terms*.

Defining Constant and Coefficient Use the information in the table to discover what a constant and a coefficient is.

Expression	Constants	Coefficients
$2a + 7 + 4a$	7	2, 4
$17 + 5 + x$	17, 5	1
$4x + 7x + 9$	9	4, 7
$22 + \frac{1}{3}z + 2y$	22	$\frac{1}{3}$, 2
$0.2q + 0.6r + 0.8s$	None	0.2, 0.6, 0.8
$4a$	None	4
3	3	None

In the expression $5 + 3x + x + 2$, Warda says that 3 and 1 are coefficients, and 2 and 5 are constants. Reda says there is only one coefficient, 3, but agrees that 2 and 5 are constants. Who is correct? Explain your reasoning.

Let's Chat Discuss the following questions with a partner:

- How would you define a constant?
- How would you define a coefficient?
- Do variables that appear by themselves have coefficients? Explain your thinking.

Identifying Parts of a Real-World Expression Now that you know what an algebraic expression is and what the parts of an expression are, you can identify the meaning of expressions that represent real-world situations.

Identifying Parts of a Real-World Expression An arcade uses both tickets and tokens with varying values.

Suppose you have the same number of tokens with different values: 10-tokens and 20-tokens. Then, you receive another 250 tickets, each with a value of 1.

You could model this situation algebraically with the expression, $10x + 20x + 250$ where x represents the number of tokens.

Identify the *terms, like terms, constants,* and *coefficients* within this expression.

Terms	(A) ______
Like Terms	(B) ______
Constants	(C) ______
Coefficients	(D) ______

E. Describe the real-world meaning of each term in the expression.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Writing Algebraic Expressions

Quick Code
egm6045

Learning Targets

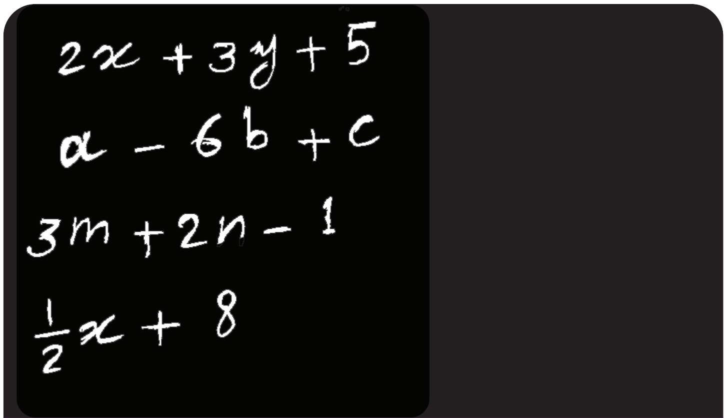
- I can use a life-size number line to act out numeric and algebraic expressions.
- I can write verbal expressions that represent numeric expressions and algebraic expressions that represent real-world scenarios.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

ACCESS

Algebraic Expressions Take a look at the photo. How can you make sense of what is on the chalkboard?



Algebraic Expressions

Let's Chat

- Do you see expressions or equations?
- Do you see numeric and algebraic expressions? Explain.
- How could you describe the difference between numeric and algebraic expressions?

BUILD AND CONNECT

Part A: Acting Out Expressions

Two Steps Forward Create a number line on the floor to act out numeric and algebraic expressions. Use this online digital resource for Hands-On exploration.

Part B: Writing Algebraic Expressions

Whiteboard: Written Expressions from Algebraic Expressions Utilize the Whiteboard to complete the following questions. Demonstrate your understanding by drawing in your journal or using the digital tool.

Addition	Subtraction	Multiplication	Division
• Increased by sum			

Operation Words Recreate a table like the one shown. In each column, list words or phrases that may be used to suggest the operation in a math problem. The first column has been started for you.

Addition	Subtraction	Multiplication	Division
• Increased by sum			
(A) ______	(B) ______	(C) ______	(D) ______

An algebraic expression can be written with words. This is called a written expression or a verbal expression.

One Operation Choose words from your list in the previous task to write each of these algebraic expressions using words. Give two different written expressions for each.

A. $x + 2$

B. $x - 5$

C. $\frac{12}{x}$

D. $12x$

E. xx

Two Operations When an algebraic expression has more than one operation, it gets more complicated to write a verbal expression.

Record all verbal expressions that represent the expression $5 + 2x$.

A. the sum of 2 and 5 multiplied by x

B. the product of 7 and x

C. the sum of 5 and the quantity 2 times x

D. the product of $2x$ and 5

E. the sum of 2 times x and 5

Algebraic Expressions from Written Expressions and Real-World Scenarios
To write algebraic expressions, assign a variable to the unknown number. Use parentheses when a sum or difference is multiplied. For example, "Four times the sum of a number and seven," translates to $4(x + 7)$, while "the sum of four times a number and seven" is $4x + 7$.

Less than and *more than* are key words for switching the order of the terms. For example, *a number less than ten* translates to "$10 - x$", while *a number less ten* is "$x - 10$".

Now, try writing algebraic expressions for the following written expressions and real-world scenarios.

Addition Record which expressions can be represented by an algebraic expression that contains addition.

A. Take 14 away from a number.

B. Ammar has 7 more tokens than Tamer.

C. A number increased by 3.5.

D. Baher put 12 more stickers in the sticker book.

E. A student shared his orange equally with his 2 friends.

Subtraction Select the algebraic expression that represents the verbal expression:
Twelve less than three groups of y.

A. $12 - 3y$

B. $y - 3(12)$

C. $3y - 12$

D. $12(3) - y$

Multiplication Two students are writing the algebraic expression for this real-world situation: "Mohamed brought 4 boxes containing c cookies to school for his birthday."

Ali writes an algebraic expression to describe the total number of cookies: "$c + c + c + c$". Kamal thinks there is another expression as well.

What expression could Kamal have been thinking of, and why are they both correct?

Division Write a verbal statement or real-world situation that could be represented by an algebraic expression that uses division.

What is the algebraic expression that models your verbal statement?

The following tasks will help you to learn about some common errors and how to avoid them.

Hazem's Try Hazem wrote "*m* added to 18 divided by 3" as the written expression for $m + \frac{18}{3}$. Is Hazem correct? Explain your reasoning.

Grandma's House Write an algebraic expression to compute the number of liters of gas Fares needs to buy to make the round trip to visit his grandma. His car gets 15 kilometers per liter. Use the variable *d* to represent the distance in kilometers to his grandma's house.


Gathering Eggs

Photo Credit: PhotoSongserm /Shutterstock.com

Gathering Eggs Maram saw this problem in her math book:

"Aunt Farha gathers the same number of eggs from her chickens every day for two weeks.

In the third week, she cooks exactly half of the eggs she had gathered previously. How many eggs does she have left?"

Which one of these options will help Maram solve this problem?

- **A.** Divide x by 2 and subtract it from 14.
- **B.** Multiply x by 14 and divide the product by 2.
- **C.** Multiply x by 14 and subtract the quotient of x and 2.
- **D.** Divide x by $\frac{1}{2}$ and subtract it from 14 plus x.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 4

Ordering of Operations and Exponents

Quick Code
egm6051

Learning Targets

- I can review the basic order of operations.
- I can simplify numeric expressions that contain exponents.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.

ACCESS

Order of Operations Does order matter?



Feteer Takeout

Suppose you are ordering a feteer. Does it matter if you ask to serve it with honey and molasses instead of molasses and honey? Does it matter if the person making the feteer adds sugar before or after baking it? In some situations, order isn't important, but at other times, it's extremely important. Can you think of some real-world situations in which order matters?

Let's Chat In previous courses, you have used the order of operations to simplify an expression that contains more than one operation. Discuss what the order of operations is with a partner.

BUILD AND CONNECT

Simplifying Numeric Expressions

Simplify an Expression Given the expression $19 - 36 + 4 \div 2$, explain how to simplify this expression using the order of operations.

Exploring Another Operation Consider how to simplify an exponential expression with a base of 10.

$$10^2 = 10 \times 10 = 100$$
$$10^3 = 10 \times 10 \times 10 = 1{,}000$$

Would you use a similar process to simplify an exponential expression with a base other than 10?

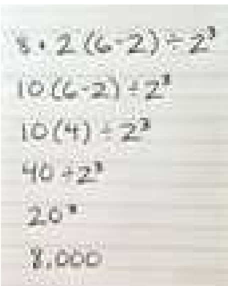
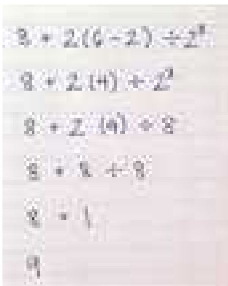
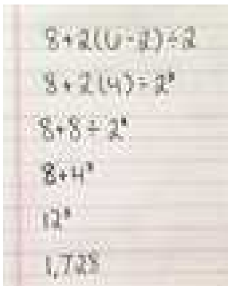
Simplify Use what you know about exponents to respond to the given questions.

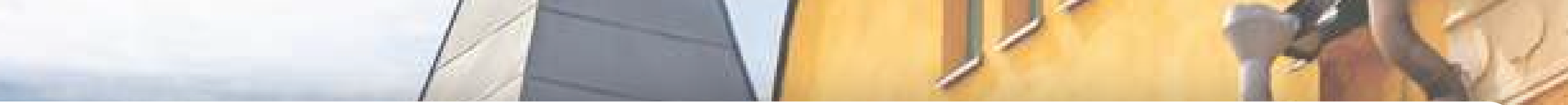
A. Describe what the expression 9^2 represents by answering each of the following:
- What is the base?
- What is the exponent?
- What is the simplified value and how did you find it?

B. Describe what the expression 2^3 represents by answering each of the following:
- What is the base?
- What is the exponent?
- What is the simplified value and how did you find it?

Let's look at some expressions that include exponents.

Three students were asked to simplify the expression: $8 + 2(6 - 2) \div 2^3$. The answers were different.

Omnia's Answer	Menna's Answer	Hadeel's Answer
 	 	



Identify the Operations Use the work shown to enter your analysis for each student.

A. In which order did Omnia perform the operations to simplify the expression?

B. In which order did Menna perform the operations to simplify the expression?

C. In which order did Hadeel perform the operations to simplify the expression?

Analyze Solutions Who do you think simplified the expression correctly? Explain your reasoning.

> **Let's Chat** Discuss with a partner who you think simplified the expression correctly. Be sure to explain your reasoning and discuss why you think the other solutions are not correct.

Exponent Expression Now that you've explored where exponents fit into the order of operations, simplify this expression:

$$(15 - 9) + 3 \times 4^2 \div 2$$

Now, order the operations yourself.

Order the Operations Place the given operations in the correct order, from first to last.

Parentheses	Add or subtract from left to right
Exponents	Multiply or divide from left to right

First	(A) ______
	(B) ______
	(C) ______
Last	(D) ______

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 5
Evaluating Algebraic Expressions

Quick Code
egm6053

Learning Target

- I can use substitution to evaluate algebraic expressions related to real-world situations.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.

ACCESS

Shopping at the Mall Look at the image of the T-shirts.


T-shirts

Photo Credit: Davdeka / Shutterstock.com

You want to buy some T-shirts. Each T-shirt is 100 LE and you have a coupon for 40 LE off your entire purchase.

Let's Chat Discuss shopping trips in which you have used a coupon. Then, discuss the T-shirt shopping scenario.

- Was the coupon for a single item or any number of items?
- How did you determine the cost of your purchase with the coupon?
- Could you use the same strategy for buying T-shirts in the scenario?

BUILD AND CONNECT

Evaluate Expressions



Buying Clothes

Photo Credit: epic_images / Shutterstock.com

Write an Expression Complete these two tasks.

A. What algebraic expression can you write to represent the situation? Explain what each symbol in your expression represents.

B. What is the cost of 4 T-shirts? Explain how you found that cost.

Let's Chat Your friend told you that he "evaluated" the expression he wrote to find the cost of 6 T-shirts. He also said he could evaluate the expression to find out the cost of 7, 8, or 9 T-shirts. How do you think he figured out what the costs were? What do you think *evaluate* means here?

HANDS-ON ACTIVITY

Evaluate Expressions Evaluate expressions and investigate how the value of an expression changes based on the value of the variable in the expression. Use this online digital resource for Hands-On exploration.

Evaluate an Expression Use your definition to complete these tasks.

A. Evaluate the expression $6 \div (8x - 3)$ when $x = 0.5$.

B. Describe the steps you took to evaluate the expression.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 6

Applications on Algebraic Expressions

Quick Code
egm6055

Learning Target

- I can evaluate expressions involving exponents and parentheses.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeating reasoning.

ACCESS

Steps in the Order of Operations What steps do you take to evaluate an expression using the order of operations?



Stairs

Example Expression

$12 - 8 \div 4 + 7 + 1 - 2^2 \times 3$
$12 - 8 \div 4 + 8 - 2^2 \times 3$
$12 - 8 \div 4 + 6^2 \times 3$
$12 - 8 \div 4 + 36 \times 3$
$12 - 2 + 108$
118

Let's Chat Consider where exponents fit into the order of operations and how to evaluate expressions.

- What is the order of operations?
- How does the order of operations change when there are exponents?
- When do you substitute the value of the variable?

BUILD AND CONNECT

Evaluating Exponential Expressions

Use what you know about the order of operations to evaluate the expression given.

Ordering the Operations What steps would you take to evaluate the expression $9 + \left(p^2 - 3\right) \div 2$ for $p = 5$?

Multiply	Add	Subtract	Divide
Simplify Exponent		Substitute the value of the variable	

Step 1	(A) ______
Step 2	(B) ______
Step 3	(C) ______
Step 4	(D) ______
Step 5	(E) ______

Whiteboard: Evaluate the Expression Utilize the Whiteboard to complete the following question. Demonstrate your understanding by drawing in your journal or using the digital tool.

Addition	Subtraction	Multiplication	Division	Exponents
* Increased by sum				

Evaluate the Expression Evaluate the expression $9 + \left(p^2 - 3\right) \div 2$ for $p = 5$, following the steps you determined in the previous question.

Let's Chat Discuss with a partner the steps you took to evaluate the expression.

- Did you both evaluate the expression in the same way?
- Do you think everyone in your class found the answer in the same way? Why or why not?

Now, try evaluating another expression.

Select the Order Which of the given orders of operations would you use to evaluate the expression $7 + 6\left(t^2 - 3\right)$ for $t = 4$? Select the appropriate order.

- **A.** multiply, simplify exponent, subtract, add
- **B.** simplify exponent, subtract, multiply, add
- **C.** add, multiply, simplify exponent, subtract
- **D.** simplify exponent, add, subtract, multiply
- **E.** simplify exponent, multiply, add, subtract

Find the Value Evaluate the expression $7 + 6\left(t^2 - 3\right)$ for $t = 4$. Record your calculations and final value.

Let's Chat Share with a partner your answers in Select the Order and Find the Value. Revise your answers as needed. Be prepared to share your findings with your peers.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 7

Determining Equivalent Algebraic Expressions

Quick Code
egm6059

Learning Target

- I can explore whether two expressions are equivalent using a balance scale as a concrete model.

Classroom Practices

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

ACCESS

Balance Scale Some algebraic expressions can look different but have the same value when evaluated.

To explore this idea, you will be using a balance scale in an activity called Keep It Level.



Balance Scale

Let's Chat

- What are your previous experiences in using a balance scale?
- Why do you think the activity might be called Keep It Level?
- How do you think a balance scale could help you determine if two expressions have the same value?

BUILD AND CONNECT

Are They Equivalent?

HANDS-ON ACTIVITY

Keep it Level Explore how to identify equivalent expressions. Use this online digital resource for Hands-On exploration.

Let's Chat If the directions asked for you to use two things of equal weight to represent both variables and constants, would any of your answers change? Why or why not?

Evaluate the following expressions using two positive integers of your choosing. Show your work as done in this example.

	$x + 2x$	$2(x + 2)$	Equal?
If $x = 1$	$1 + 2(1)$ $1 + 2$ 3	$2(1 + 2)$ $2(3)$ 6	no
If $x = 2$	$2 + 2(2)$ $2 + 4$ 6	$2(2 + 2)$ $2(4)$ 8	no

Evaluate for Equivalency 1 Evaluate each of these expressions using two different positive integers of your choosing.

If the expressions are equal, answer *yes*. If they are not equal, answer *no*.

	$3(2x + 1)$	$6x + 3$	Equal?
If $x =$ (A) ______	(B) ______	(C) ______	(D) ______
If $x =$ (E) ______	(F) ______	(G) ______	(H) ______

Evaluate for Equivalency 2 Evaluate the given expressions using two different positive integers of your choosing.

If the expressions are equal, answer *yes*. If they are not equal, answer *no*.

	$3x + 6$	$x + 3 + 2(x + 1)$	Equal?
If x = (A) ______	(B) ______	(C) ______	(D) ______
If x = (E) ______	(F) ______	(G) ______	(H) ______

Evaluate for Equivalency 3 Evaluate these expressions using two different positive integers of your choosing.

If the expressions are equal, answer *yes*. If they are not equal, answer *no*.

	$4x + 6$	$3(x + 2)$	Equal?
If x = (A) ______	(B) ______	(C) ______	(D) ______
If x = (E) ______	(F) ______	(G) ______	(H) ______

Let's Chat Is it possible for the value of two expressions to be equal sometimes and not others?

Are They Equivalent? Examine these two expressions and determine whether they are equal. If so, consider whether they are always equal. Complete each task.

$$2(x + 1) \qquad 2x + x$$

A. Try to find a value for x that will make these expressions equal.

B. Try to find a value for x that will make the expressions not equal.

C. Decide if these two expressions are always equal and if they should be considered equivalent expressions.

Let's Chat Share with a partner your answers in Are They Equivalent?. Revise your answers as needed. Be prepared to share your findings with your peers.

Check Your Understanding Follow your teacher's instructions to complete this activity.

UNIT 4

Theme 1 | Number Sense and Operations: Expressions and Equations

Equations and Inequalities



Video

Equations and Inequalities

Unit Opener Video: Equations and Inequalities

Quick Code egm6067

Use geology and signage to present scenarios in which mathematical equations can be used to answer real-life questions.

Key Vocabulary

algebraic equation, constraints, inequality, inverse operations, like terms, solution

Photo Credit: (t) Veranika848 / Shutterstock.com; (b) Anton Petrus / Shutterstock.

LESSON 1

Solving Algebraic Equations

Quick Code
egm6071

Learning Target

- I can use a pan balance to model and solve algebraic equations.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.

ACCESS

Hands-On Activity: Keep It Equal This Hands-On Activity can be used for modeling equations and can be used for practice. Use this online digital resource for Hands-On exploration.

Using a Pan Balance to Solve Equations As you explore the pan balance, consider what happens when you add x blocks compared to adding 1 block onto the scale.



Pan Balance

Photo Credit: Shutter_M / Shutterstock.com

BUILD AND CONNECT

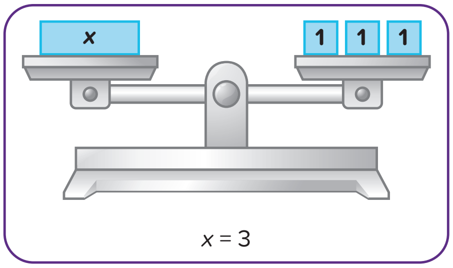
Quick Code
egm6071

Pan Balance Interactive

Interactive

Go online to explore this interactive activity.

Model and Solve Complete these three tasks.



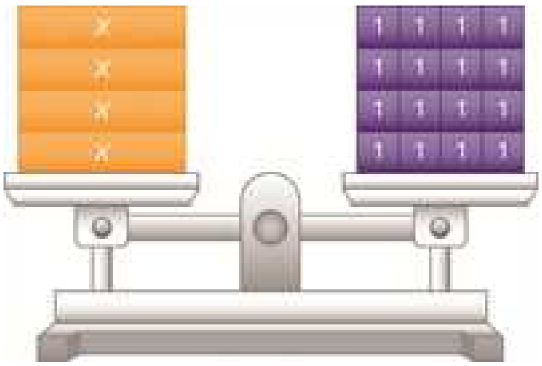
A. Explain the difference in modeling these two equations on a pan balance.

$3x = 9$ and $x + 8 = 17$

B. Explain how you would use the balance to find x for the equation $3 + x = 12$.

C. Explain how you would use the balance to find x for the equation $6x = 18$.

Write an Equation and Solve Answer the following questions.



A. Write an equation for the previous model. Explain how you wrote the equation.

B. Solve the equation. How do you know that you solved the equation correctly?

Let's Chat Now that you've had the chance to explore solving equations with the balance, can you think of a way to solve an equation like $x + 5 = 11$ without the tool, but using the idea of keeping the sides of the equation balanced? Discuss with a partner.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 2

Exploring Inequalities

Quick Code
egm6079

Learning Targets

- I can explore signs that indicate a restriction such as a speed limit, a sale markdown, a weight limit, or a capacity limit.
- I can analyze these scenarios and determine how they differ from equation scenarios.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and make use of structure.

ACCESS

Exploring Inequalities In everyday life, you see a variety of signs on streets, in stores, in apartment buildings, and many other places. Let's see if you can interpret the signs in the following scenarios. As you work through each of the scenarios, think about how they are similar to one another and how they are different.



Street Signs

Let's Chat What are the signs that you see in everyday life? How are they similar to and different from one another?

BUILD AND CONNECT

Road, Sale, Height, and Weight Signs

Road Sign The sign shows the speed limit for a road in kilometers per hour. Record all speeds that are acceptable to drive on the road.

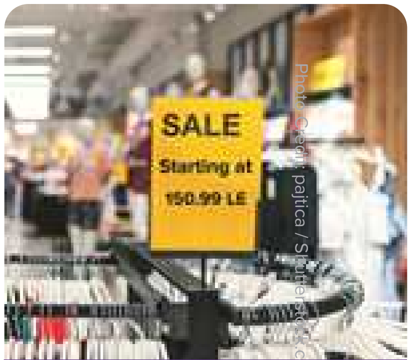
A. 38 km/hr
B. 50 km/hr
C. 30 km/hr
D. 40 km/hr
E. 43 km/hr
F. 49 km/hr



Speed Limit

Sale Sign The sign shows the sale prices of some clothing on a sale rack. Use the sign to determine any prices you might expect to pay for an item from this rack. Record all prices that apply.

A. 140.99 LE
B. 180.99 LE
C. 150.49 LE
D. 290.99 LE
E. 120.99 LE
F. 150.99 LE



Sale Prices

Height Sign The sign shows the height limits for an amusement park ride.

A. List three acceptable heights for a person to be able to ride the roller coaster.
B. List three unacceptable heights for a person to be able to ride the roller coaster.



Ride Height Resctrictions

Weight Sign The sign shows the weight limits for vehicles stowed (stored) on a ramp and vehicles transiting (traveling across) the ramp.

> **CAUTION**
>
> **Weight Restrictions**
>
> The total weight of vehicles stowed on the ramp shall not exceed 47,000 kg
>
> Vehicles transiting on the ramp shall not exceed 24,500 kg
>
> **Weight Limit**

Answer these questions.

A. Suppose 3 vehicles are stowed on the ramp at the same time. What are some possible weights of the 3 vehicles?

B. Now, suppose 3 vehicles are traveling across the bridge. What are some possible weights of these 3 vehicles?

Let's Chat Look back at all four scenarios and discuss the following with a partner:

- Can any of the situations be modeled by an equation?
- Why or why not?

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Solving Inequalities

Quick Code
egm6081

Learning Target

- I can use a number line to represent inequalities.

Classroom Practices

- Reason abstractly and quantitatively.

ACCESS

Exploring Inequalities on a Number Line In this lesson, you will be using number lines to represent inequalities. Think about other ways in which you have used number lines to represent numbers, such as identifying rational numbers or solving problems.

Let's Chat Discuss how you could show a range of numbers on a number line.

BUILD AND CONNECT

Solution of an Inequality Use this number line to investigate how inequalities look.

Quick Code
egm6081

Interactive

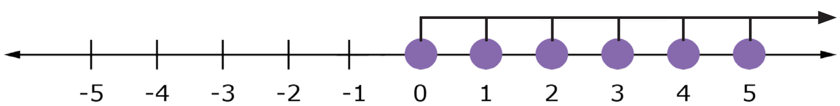
Representing Inequalities on a Number Line Interactive

Go online to explore this interactive activity.

Solutions of inequalities in the set of integers can be represented as follows:

$x > -1$

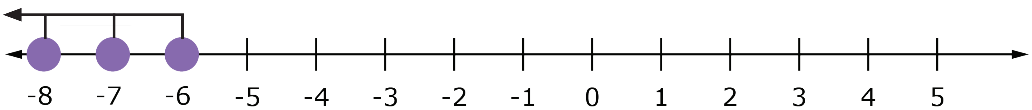
a .



Does -1 belong to the solution set of the above inequality? explain that.

$x \leq -6$

b .



Does -6 belong to the solution set of the above inequality? explain that.

- The values of the variable x that satisfy the inequality (a) in the set of rational numbers are all rational numbers greater than -1
- The values of the variable x that satisfy the inequality (b) in the set of rational numbers are all rational numbers smaller than or equal -6

Let's Chat How are these number lines different from ones you have used before? Consider these algebraic statements to prepare for discussion.

$x > -6$	$x \geq -6$	$x = -6$
$x < -6$	$x \leq -6$	

Let's Chat Think about how you would read each statement, then discuss the following with a partner.

- What would each inequality look like on a number line?
- Which way would each graph point? Why?

Comparison 1 Record each true statement about the graphs of $x > -2$ and $x < -2$ on the number line.

A. -2 belongs to the solution set of each of them.
B. -2 belongs to the solution set of only one of them.
C. The inequality $x > -2$ includes all values to the left of -2 on the number line.
D. The inequality $x < -2$ includes all values to the right of -2 on the number line.
E. They have no points in common.

Comparison 2 Record each true statement about the graphs of $x > -2$ and $x \geq -2$ on the number line.

F. -2 belongs to the solution set of each of them.
G. -2 belongs to the solution set of only one of them.
H. The inequality $x \geq -2$ includes all values to the left of -2 on the number line.
I. The inequality $x > -2$ includes all values to the right of -2 on the number line.
J. They have no points in common.

Make a Conjecture How will the graphs for each pair of algebraic statements be similar? How will the graphs be different?

A. $x < -2$ and $x \leq -2$
B. $x \leq -2$ and $x \geq -2$
C. $x = -2$ and $x > -2$

Now that you've explored inequalities on the number line, think about what it means for a point to be included on the graph of an **inequality**. For example, would 2 be on the graph of $x \leq -1$?

Let's Chat Discuss with a partner. Inequalities have an infinite set of solutions. The solution set for $x < 4$ would be every number less than 4. How would $x < 4$ be represented on a number line?

Define the Solution of an Inequality In your own words, explain what you think a solution of an inequality is. How is the graph of an inequality related to the solution of an inequality? Record your answer.

Think about how you can use the definition of the solution of an inequality or its graph on the number line to determine if a given value is a solution of an inequality. Also, think about how many solutions an inequality has.

Find Solutions Determine whether each of the given values are solutions of $x < 9$. Record all the values that are solutions.

A. -9
B. 10
C. 0
D. 2.4
E. 9.1
F. -0.9
G. 8.9
H. -6
I. 15

Check Your Understanding Follow your teacher's instructions to complete this activity.



UNIT

5

Theme 2 | Mathematical Operations and Algebraic Thinking: Statistics and Data Analysis

Dependent and Independent Variables

Video



Dependent and Independent Variables

Unit Opener Video: Dependent and Independent Variables

Quick Code
egm6087

Explore independent and dependent variables in the world around you.

Key Vocabulary

dependent variable, independent variable

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LESSON 1

The Relationship Between Dependent and Independent Variables

Quick Code
egm6089

Learning Targets

- I can explore and apply formal definitions of **independent variables** and **dependent variables**.

Classroom Practices

- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.

ACCESS

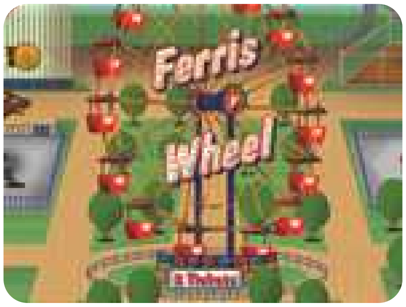
Let's Go to the Amusement Park Have you ever been to an amusement park? At most amusement parks, you can go on rides, play games, see animals, and buy snacks at food stands.



Amusement Park

Explore the pictures of the amusement park. What mathematical relationships can you see at the amusement park?









Let's Chat Discuss with a partner any mathematical relationship that you notice from the information in the images.

BUILD AND CONNECT

Part A: Identify and Explore Relationships

Identify Relationships Use the amusement park information from the images as you complete the following tasks.

A. Describe two mathematical relationships among the different parts of the amusement park.

B. Choose one of these relationships to explain how one quantity depends on another.

Explore a Relationship Represent the cost (in tickets) of riding the roller coaster using two or more ways: a table, graph, model, diagram, or an expression.

Explain how you can use your representations to figure out how many tickets are needed to ride the roller coaster 8 times.



Let's Chat Discuss your representations with a partner. How were they similar? How were they different?

Dependency When you explored the diagram of the amusement park, you saw that in some of the relationships, one quantity depended on another. Let's explore dependency a little bit more.

Analyze Phrases Complete these statements by filling in each blank with a given phrase to show which depends on the other. You should use all phrases to create three different dependencies.

how hard you laugh	what is on the menu
what you spend your tickets on	how funny a joke is

____(A)____ depends on the kinds of rides you like.

What you order at a food stand depends on ____(B)____.

____(C)____ depends on ____(D)____.

Let's Chat Some of these phrases are dependent and others are independent phrases.

- What do you think the words *dependent* and *independent* mean here?
- Which part of the sentences you completed are independent?
- Which are dependent? Discuss these terms with a partner.

Think back to the roller coaster example. Which quantity depends on the other: the number of tickets you need or the number of times you want to ride the roller coaster? How do you know?

Part B: Dependent and Independent Variables

You can represent phrases that can be quantified with variables. Suppose t represents the number of tickets you need, and r represents the number of times you want to ride the roller coaster. Which variable depends on the other?

An independent variable is a variable whose value is not determined by any other value or variable, so it can be assigned any value. The variable that changes depending on the value of the independent variable is called the dependent variable.

In the roller coaster situation, which is the independent variable and which is the dependent variable?

Write Statements Complete the statements. Be sure to name a variable to represent each quantity you fill in and be prepared to explain how you know which variable is independent and which is dependent.

1. The number of balloons, b, you buy depends on ____(A)____.
2. ____(B)____ depends on how much time t you can spend at the amusement park.
3. ____(C)____ depends on ____(D)____.

Let's Chat Discuss your answers to the last exercise with a partner.
Which variable is dependent in each of your statements? Which is independent? Explain how you know.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 2

Applications on Dependent and Independent Variables

Quick Code
egm6091

Learning Target

- I can examine and create algebraic equations to model real-life situations.

Classroom Practices

- Reason abstractly and quantitatively.
- Model with mathematics.

ACCESS

Think back to the relationship you explored involving the roller coaster at the amusement park.

Explore an Equation An equation to represent the relationship is $t = 6r$. Write which of the given choices should be used to define each description.

r t 6

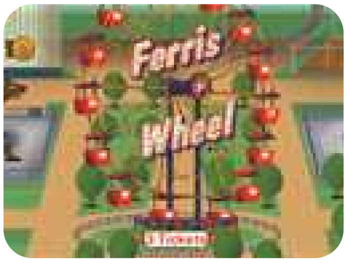
(A) ______ Number of times you want to ride the roller coaster

(B) ______ Total number of tickets your need

(C) ______ Number of tickets needed to ride the roller coaster one time

BUILD AND CONNECT

Write and Use an Equation Now, go back to the amusement park. This time, go to the rides area, and explore the other rides.











Suppose a student chose a ride and said that for her ride, the number of tickets, t, she would need to go on her ride r times could be represented by $t = 5r$.

Which ride did she choose? How can you use her equation to find how many tickets she would need if she wanted to go on this ride 15 times?

The student says that she now understands why she needs to learn about dependent and independent variables. She can see in the equation $t = 5r$, that the value of r depends on the value of t.

Explain whether she is correct.

Let's Chat Discuss your answers with a partner.

Ride	Number of Tickets
Ferris Wheel	3 Tickets
Rock N Roller	6 Tickets
Scrambler	8 Tickets
Swing	4 Tickets
Viking	5 Tickets

Write an Equation Now, select a different ride. Think about the relationship between the number of tickets you need for the new ride and the number of times you want to ride. Write an equation to represent the situation using the variables r and t.

Use Your Equation How can you use your equation to find the number of tickets you would use to go on your ride 12 times? Explain your reasoning.

Analyze Another Situation Now, look for mathematical relationships at the food stand.



Popcorn Equation Think about a relationship between the food you want to buy and the amount of tokens you want to spend. Suppose you are buying 1 box of popcorn for a friend and you are not sure what to buy for yourself. Determine what is known and what will change.

Select two variables to write an equation for the total amount of tokens you will spend at the food stand and explain what each part of the equation represents.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Analyzing The Relationship Between Dependent and Independent Variables

Quick Code
egm6093

Learning Target

- I can discover how to use independent and dependent variables when writing equations.

Classroom Practices

- Model with mathematics.

ACCESS

Input and Output Have you ever thought about how certain outcomes directly impact the outcome of future events? At the amusement park, you and your friends get a bag of mixed snacks. If you are reaching into the bag and choosing a snack, do your options change after each choice?



Trail Mix

Let's Chat Discuss your answer with a partner:
As you take turns choosing snacks, what do you think happens to the options in the bag?

HANDS-ON ACTIVITY

What's the Rule? In this Hands-On Activity, you will figure out rules using input and output. Use this online digital resource for Hands-On exploration.

BUILD AND CONNECT

Rules, Variables, and Equations

Rules Can you always figure out the correct rule with one input and output number? Why or why not?

Variables Use the equation $y = 3x$ to answer these questions.

A. Which variable represents the input number?
B. Which variable represents the output number?
C. Which is the dependent variable?
D. Which is the independent variable?

Show What You Learned Complete each statement, using the variables x and y as needed.

1. If the rule is "Multiply by 2," you would write the equation as: ___(A)___.
So, if $x = 2.3$, then y would be: ___(B)___.

2. If the rule is "Add 6," the equation is: ___(C)___.
So, if x is $\frac{1}{4}$, then y would be: ___(D)___.

Let's Chat Equations can have more than one operation. Talk with a partner about how you predicted which input and output numbers would probably have two operations.

Write an Equation Use the variables x and y, where x is the independent variable. Write the equation for "multiply by 3 and add 4." Select the correct answer.

A. $x = y + 3x + 4$
B. $x = 3y + 4$
C. $y = 3x + 4$
D. $y = -3 + 4x$

Equation from a Table Use variables x and y to write the equation for each of the tables.

A.

x	0	4	8	12
y	4	8	12	16

B.

x	12	20	8	4
y	7	11	5	3

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 4

Graph Representation for Dependent and Independent Variables

Quick Code
egm6095

Learning Targets

- I can connect representations of dependent and independent variables in tables, equations, and graphs.
- I can represent dependent and independent variables on a coordinate plane.

Classroom Practices

- Model with mathematics.

ACCESS

Making Graphs In previous lessons, you explored the rides area and the food stand at the amusement park. Now, let's explore the games. Choose one of these games.







Let's Chat What information are you given about the game?

BUILD AND CONNECT

Part A: Tables, Equations, and Graphs

Make a Table Choose one of the three games. Create a table that shows the number of objects you buy and the amount of money it would cost for those objects. Let x be the number of rings, balls, or darts, depending on the game you chose, and let y be the total cost.

Ring Toss	Balloon Pop	Basketball
4 Rings for 5.00 tokens	3 Darts for 4.50 tokens	2 Balls for 5.00 tokens
Rings also sold individually!	*Darts also sold individually!*	*Balls also sold individually!*

Game Choice: (A) ______

x	(B) ______	(D) ______	(F) ______	(H) ______
y	(C) ______	(E) ______	(G) ______	(I) ______

Let's Chat Discuss your table with a partner. What did you need to know in order to create your table?

Write an Equation Record your responses for each of these two tasks.

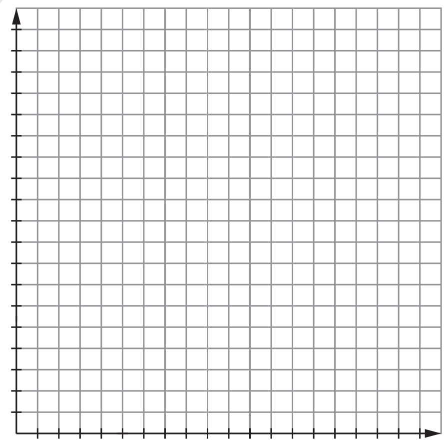
A. Write an equation that represents your table, using x and y as variables to find the total cost of playing the game for any number of rings, balls, or darts.

B. Identify the independent and dependent variables, and explain what each part of your equation means.

Let's Chat Now that you have made a table, think back to when you graphed relationships involving ratios. Discuss with a partner how you might graph the relationship for the game you chose.

- On which axis would you put the independent variable? Why?
- On which axis would you put the dependent variable? Why?

Whiteboard: Make a Graph Make a graph based on the plan you made with your partner. Be sure to give your graph a title and label the axes x or y accordingly. Demonstrate your understanding by drawing in your journal or using the digital tool.



Submit Your Graph Explain how you made your graph. Remember to include what each axis represents.

Let's Chat Find a partner who chose a different game, and compare your tables and graphs.

Part B: Analyze Another Situation

How Many Rides You go on three fewer rides than your friend because she got to the amusement park earlier than you.

Explain what the variables x and y mean in this situation.

A. What does x describe?

B. What does y describe?

Then, create a table like the one shown to enter some values of x and y to represent the number of rides you and your friend have been on at different times during your amusement park visit.

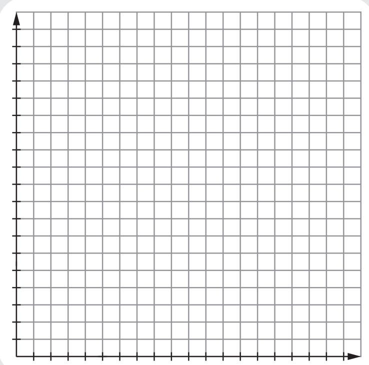
x	(C) ____	(E) ____	(G) ____	16
y	(D) ____	(F) ____	(H) ____	(I) ____

Let's Chat Discuss your table with a partner. What does each variable represent?

Ride Equation Write an equation to represent the relationship in the table you made. Identify the independent and dependent variables and explain what each part of your equation means.

Let's Chat Now that you've made a table and written an equation, discuss with a partner how you might graph this relationship. On which axis would you put the independent variable? How about the dependent variable? How did you decide?

Whiteboard: Ride Graph Make a graph based on the plan you made with your partner. Be sure to give your graph a title and label the axes x or y accordingly. Demonstrate your understanding by drawing in your journal or using the digital tool.



Ride Graph Sketch the graph with an explanation about how you made it. Be sure to define what each axis represents.

Let's Chat Discuss with a partner the following question:

- How was this graph similar to the one you made to represent the games? How was it different?

Check Your Understanding Follow your teacher's instructions to complete this activity.



Photo Credit: EugeniaSh / Shutterstock.com

UNIT

6

Theme 2 | Mathematical Operations and Algebraic Thinking: Statistics and Data Analysis

Data Distributions

Video



Data Distributions

Unit Opener Video: Data Distributions

Quick Code
egm6101

Explore how graphs can be used to provide simplified representations of complex data.

Key Vocabulary

bar graph, box plot, categorical data, clusters, dot plot, gaps, histogram, interval, lower quartile, maximum, median, minimum, numerical data, outlier, peaks, range, scale, skewed (distribution), skewed left, skewed right, spread, statistical question, symmetrical (distribution), upper quartile, variability

MINISTRY OF EDUCATION AND TECHNICAL EDUCATION

LESSON 1

Data and Statistical Questions

Quick Code
egm6103

Learning Targets

- I can explore statistical questions and data.
- I can compare and contrast statistical and non-statistical questions.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.

ACCESS

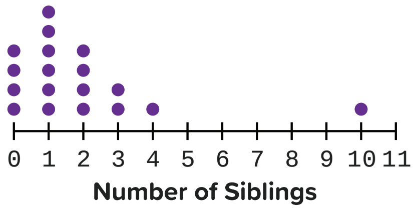
Time Capsule As he was walking to school, Diaa tripped over something in the schoolyard. After a little bit of digging, he and his classmates uncovered a time capsule that had been buried by a class of students from the past. You are about to examine five items the class found in the time capsule.



Class Photo

Time Capsule Findings

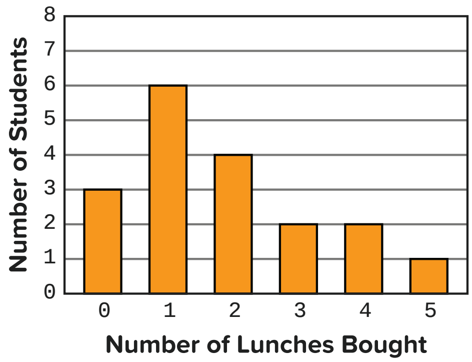
Number of Siblings per Student



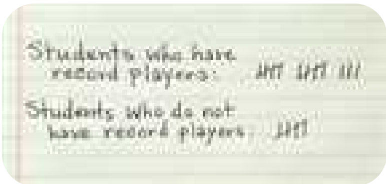
Flying in a Plane



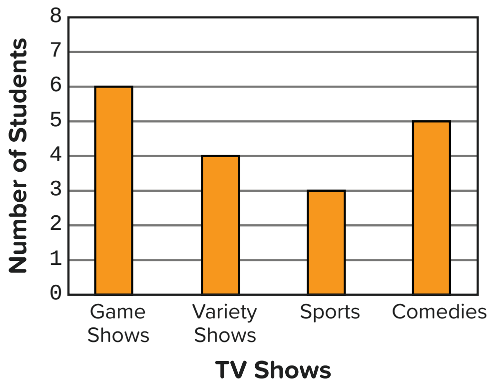
Lunches Bought per Week



Owning Record Players



Favorite TV Shows



Analyze the Time Capsule What information can you learn from looking at the time capsule? List at least three examples.

Write Questions What are three questions that the students who collected data for the time capsule may have asked?

BUILD AND CONNECT

Part A: Sort the Data

Now that you've explored the possible questions the students asked, take a look back at the different types of data that the questions produced. How can you sort the data from the time capsule?



Whiteboard: Categorizing Data Define two categories to sort the data into, and then sort the tiles according to those categories. Demonstrate your understanding by drawing in your journal or using the digital tool.

Sort the Data What categories did you use for your sort?

Think about how the questions the students asked affected the data they put into the time capsule. For example, how would the data collected be different if they asked the question, "Do you have siblings?" as opposed to "How many siblings do you have?"

In Part B, you will see how the question you ask can generate data that will provide a rich statistical discussion.

Part B: Make a Time Capsule

Time Capsule Suppose your class wants to make a time capsule that you plan to open at your 25-year reunion. What kinds of data might you want to put in a time capsule?

You want to gather data about your class to put in the time capsule, but your teacher tells you that you can only ask statistical questions to gather data about your class.

What is a statistical question? Some examples of statistical questions, as well as questions that are not statistical, are shown in this table.

Statistical Questions	Not Statistical Questions
What are the students' favorite colors?	Do you like the color red?
How many people do each of the students in the class have in their families?	How many students are in the class?
How many emails do the students in the class write per week?	What is the name of your school?
How many books do the students in the class read per year?	How many books did you read this past year?

Let's Chat Discuss your thoughts with a partner.

- What do you notice about the statistical questions as opposed to the questions that are not statistical?
- What is a statistical question?

Write Statistical Questions Use what you have learned to write two examples of a statistical question and two examples of a non-statistical question. Create a table like the one shown and create two questions of each type.

Statistical Questions	Non-Statistical Questions
(A)	(B)
(C)	(D)

Define a Statistical Question Which of these descriptions do you think most completely defines a statistical question?

A. A question with exactly three possible answers
B. A question that produces lots of different possible answers
C. A question that produces a single non-numerical answer
D. A question that can be answered with “yes” or “no”

Types of Statistical Questions Different types of statistical questions are: numerical and categorical. Think about what each of the words mean and what a numerical statistical question might be versus a categorical one.

Analyze Data from Statistical Questions Determine whether the results from each question would give you numerical data or categorical data.

Numerical Data	Categorical Data

A. How many letters do the students in your class have in their first names?
B. What are the favorite colors of students in your class?
C. What types of movies do students in your class like best?
D. What color eyes do students in your class have?
E. How many people do the students in the class have in their families?
F. What TV shows do students in your class like?
G. What are the scores on all of your math tests this marking period?
H. How many pets do the students in your class have?

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 2

Exploring the Histogram

Quick Code
egm6105

Learning Targets

- I can explore and discover characteristics of **histograms**.

Classroom Practices

- Model with mathematics.

ACCESS

Going to the Movie Theater Lots of people head to movie theaters on evenings or weekends to see thrilling adventures, touching dramas, or scary stories.


Movies

Photo Credit: Fer Gregory / Shutterstock.com

Let's Chat

- Do you enjoy going to the movies?
- Do you go with family or friends?
- What are your favorite types of movies?

BUILD AND CONNECT

Dot Plots, Bar Graphs, and Histograms

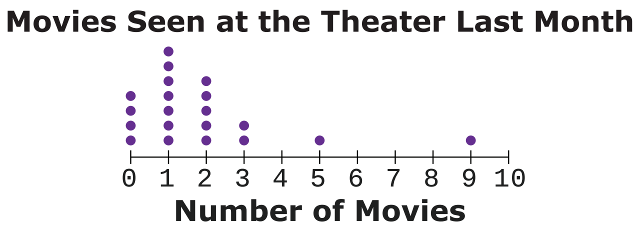
Examining Dot Plots Amira's class collected some information about people going to movie theaters. She used dot plots to display the information she collected.

Make a Conjecture Consider what you know about dot plots as you complete the tasks.

A. Create a statistical question that Amira could have asked about this topic to create a dot plot.

B. Predict what the dot plot might look like based on your statistical question.

Analyze a Dot Plot Here is one dot plot that Amira made. Record your responses to each question.



A. How many people were surveyed?

B. What other information can you determine from the graph?

Identify Characteristics There are certain characteristics that all dot plots share. Identify which of these characteristics are common to all dot plots.

A. Dot plots should have titles.

B. Dot plots should have data graphed above a number line.

C. Dot plots should have exactly 20 pieces of data.

D. Each individual piece of data can be seen on a dot plot and is represented by a dot.

E. The number lines in dot plots should start at 0.

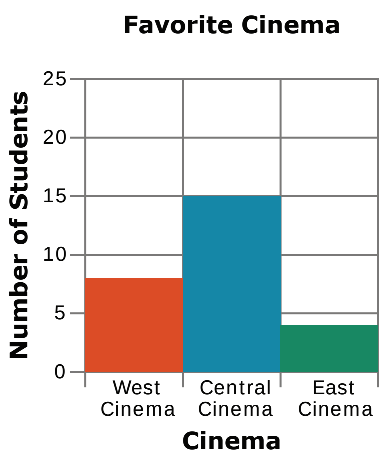
F. The number lines in dot plots should be labeled with the units used to measure the data.

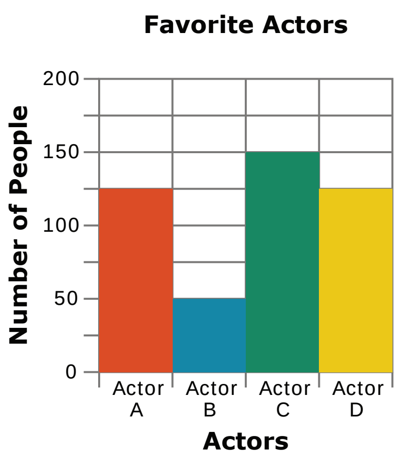
G. You can determine the number of observations in a dot plot by finding the sum of the numerical value represented by each dot.

H. Only numbers that have data points should be marked on the number line.

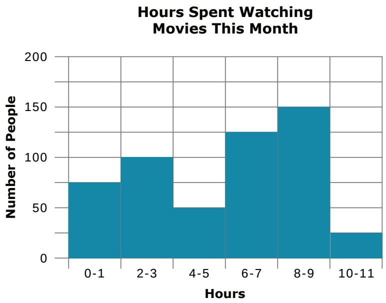
Now, let's explore some other types of graphs.

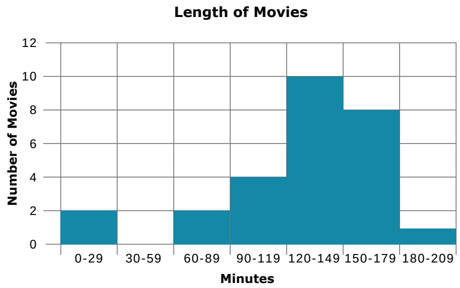
Bar Graphs and Histograms Suppose Amira's class constructed these bar graphs to show some other data they collected from different groups of people about their movie-watching habits.





Examining Histograms Amira's class also constructed some histograms to represent some of the data they collected. See if you can determine how a histogram is constructed by analyzing the next two graphs.

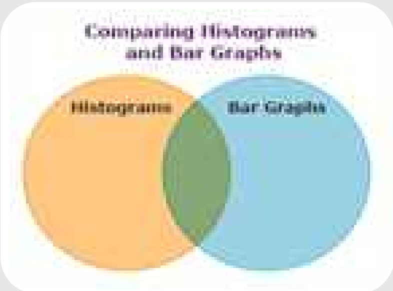




Let's Chat

- Why do you think Amira's class used histograms instead of bar graphs to represent each set of data?
- How are the histograms similar to each other? How are they different?
- What units are used in each histogram?
- Can you tell how many observations Amira's class collected for each histogram?

Whiteboard: Comparing Histograms and Bar Graphs Fill in the Venn diagram to compare bar graphs and histograms. Demonstrate your understanding by drawing in your journal or using the digital tool.



Comparing Histograms and Bar Graphs Sketch a copy of your Venn diagram.

Identify Characteristics of a Histogram There are certain characteristics that all histograms share. Identify the characteristics that are common to all histograms.

A. Histograms should have a title, and each axis should be labeled.
B. Histograms show individual data points.
C. Histograms show data grouped into intervals.
D. The bars on a histogram should touch.
E. The intervals on a histogram do not have to be the same size.
F. The intervals in a histogram should have no gaps or overlaps in values.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Representing Data Using Histograms

Quick Code
egm6107

Learning Targets

- I can create a histogram for a given set of data.
- I can collect data using number cubes and create a histogram for that data.

Classroom Practices

- Model with mathematics.
- Attend to precision.

ACCESS

Measuring a Tree A Primary 6 class decided to take care of some trees in their neighborhood. They wanted to find trees that students could measure by using their arm spans. They started by asking this statistical question: “What is your arm span?”

Let’s Chat Discuss with a partner how these students may have measured their arm spans.

BUILD AND CONNECT

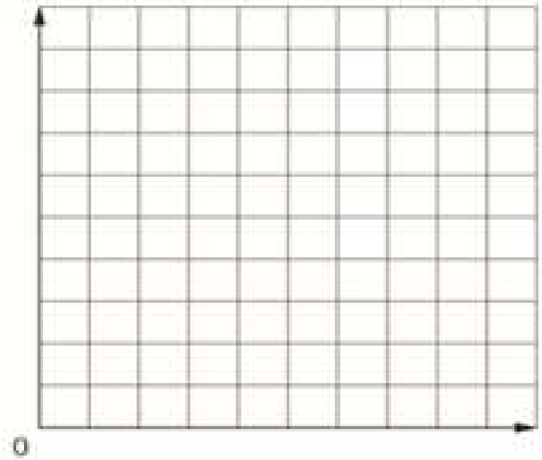
Make a Histogram Here is the data that the students collected in a frequency table. Use the Whiteboard or graph paper to create a histogram to display this data. Be sure to choose an **interval** size that makes sense for this data set.

Arm Span (cm)	Frequency
127	2
132	3
135	1
138	1
141	1
142	2
143	1
144	2

Arm Span (cm)	Frequency
147	2
149	2
152	3
153	2
157	2
158	3
160	1
166	1

Whiteboard: Create a Histogram Create a histogram to display the data. Be sure to choose an interval that makes sense for this data set. Demonstrate your understanding by drawing in your journal or using the digital tool.

Arm Span (cm)	Frequency
127	2
132	3
135	1
138	1
141	1
142	2
143	1
144	2
147	2
149	2
152	3
153	2
157	2
158	3
160	1
166	1



Arm Span Histogram Explain why you chose the intervals you used. Compare your histogram with a partner's.

Tree Measurements Using the information gathered about students' arm spans and the circumferences of species of trees given, determine which trees students could measure using arm span.

Maple: 127 cm   Oak: 182.9 cm   Dogwood: 91.4 cm

Sassafras: 137.2 cm   Ash: 148.6 cm

HANDS-ON ACTIVITY

Number Cube Sums In this Hands-On Activity, make a histogram of data that you collect yourself. Use this online digital resource for Hands-On exploration.

Number Cube Sums Answer the following questions.

A. Why might a class want to collect data for sums of number cubes? What statistical question could be answered by this data collection?

B. What information can you get by looking at the histograms your class made for this data?

More Data If you combined the data for your whole class, how do you think the histogram would change?

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 4

Exploring Box Plot

Quick Code
egm6109

Learning Target

- I can calculate the **median** and 5-point summary of a data set and describe how these values are represented on a **box plot**.

Classroom Practices

- Model with mathematics.
- Attend to precision.

ACCESS

Random Numbers Consider the numbers in the picture when answering the Let's Chat questions.



Random Numbers

Photo Credit: romeovip_md / Shutterstock.com

Let's Chat

- When you gather data, are the numbers in order or out of order?
- What are some ways in which you can order the data?
- How does ordering the data help you understand it?

BUILD AND CONNECT

Part A: Find the Median

Some graphs may give us a clearer picture of data than others. A box plot is one of those graphs. In order to explore box plots, we need to have an understanding of median.

Your teacher wants to determine how many hours a student spends doing independent math work during the school week. Gather data from your own class or use the data provided to create a dot plot to organize this data.

Number of hours students spend doing independent math work during the school week sample data.

7, 3, 8, 7, 9, 8, 2, 4, 5, 0, 2, 1, 6, 4

Whiteboard: Independent Math Hours Using the study hours data, create a dot plot, marking needed values along the number line and drawing a vertical line on the number line where you think the middle of the data might be. Then, examine the lower half of the data and draw a vertical line where you think the middle of the lower half of the data might be. Do the same with the upper half of the data. Demonstrate your understanding by drawing in your journal or using the digital tool.



Dividing the Data Use your dot plot to answer these questions.

A. How many sections of the number line were created when you drew the vertical dividing lines on your data?

B. Approximately how much of the data does each section represent?

When a set of data is ordered from least to greatest, the middle value is known as the median. By finding the median, you can determine a typical value of the data set.

Let's Chat Discuss with a partner the strategy you would use to find the median of a data set with an even number of data points.

Finding the Median Write the median for each set of data.

A. 1, 2, 3, 5, 7 **B.** 1, 2, 3, 3, 5, 7 **C.** 1, 2, 2, 3, 5, 7

Correct? Ayman is looking for the typical number of points that the players on his basketball team score each game. He records the following values for the last game and determines that the median is 3.5. Is he correct? Explain why or why not.

Each player's points in a basketball game: 1, 12, 6, 6, 5, 2, 0, 10, 7, 2

Human Number Line For further practice finding the median, complete the Hands-On Activity: Human Number Line. Use this online digital resource for Hands-On exploration.

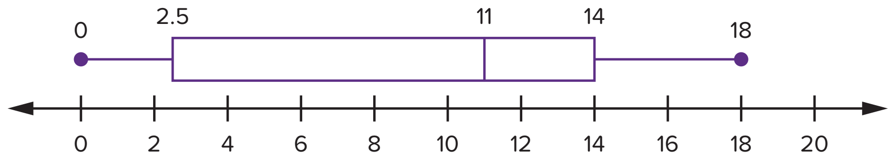
Part B: Box Plots

Quick Code
egm6109

Box Plot Maker Interactive Interactive

Go online to explore this interactive activity.

Box plot example
0, 1, 7, 4, 18, 12, 0, 9, 12, 11, 13, 17, 15



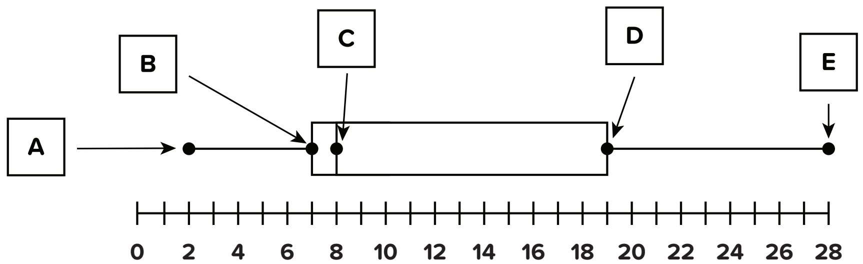
Let's Chat Discuss with a partner the important characteristics of a box plot.

Box Plot Complete the following tasks. With your peers, use the Box Plot to solve the questions.

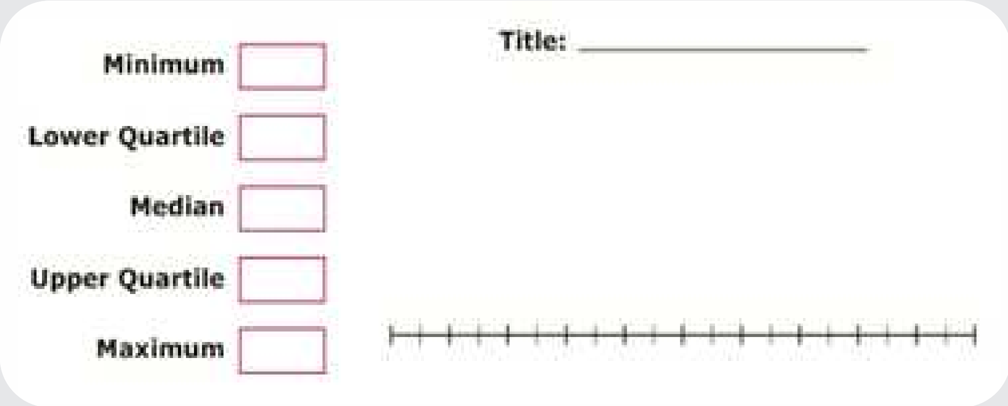
- **A.** What is the value of your minimum data point?
- **B.** What is the value of your maximum data point?
- **C.** What is the value of the median of your data set?
- **D.** Explain what feature in the box plot shows the location of the median of the data set.
- **E.** What does the data displayed in the entire rectangular box found in the middle of the data set represent?
- **F.** The endpoints of the rectangular box are called the 1st quartile (Q1) and the 3rd quartile (Q3). Use the box plot and the data values to explain the meaning of Q1 and Q3.
- **G.** The two segments from the minimum value to Q1 and from Q3 to the maximum value are sometimes called whiskers. What does each whisker represent?

5-Number Summary A data set can be described using five values, called a 5-number summary. Identify the correct term for each of the 5 points shown on the box plot.

Minimum | Maximum | Median | Upper Quartile | Lower Quartile



Whiteboard: Create a Box Plot Use the Whiteboard to complete the next questions. Demonstrate your understanding by drawing in your journal or using the digital tool.



Create a Box Plot Create your own set of data with 8–10 data points. Make a box plot representing your data. In order to fit it on the number line, the difference between the highest and lowest values should not be more than 20. When you have finished, use the interactive to check your work.

In Conclusion Identify all true statements.

A. A box plot allows you to see how many data points you have.

B. The box on a box plot shows where half of the data is located.

C. The lower quartile shows where the lower half of the data is located.

D. The upper quartile separates the lowest 3 quarters of the data from the highest quarter of the data.

E. The lower quartiles separate the first quarter and second quarter of the data.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 5

Applications on Data Representations

Quick Code
egm6119

Learning Target

- I can analyze data displays to determine which is most appropriate when answering statistical questions.

Classroom Practices

- Model with mathematics.
- Attend to precision.

ACCESS

You know how to make and analyze data using dot plots, histograms, and box plots.



Analyze Data

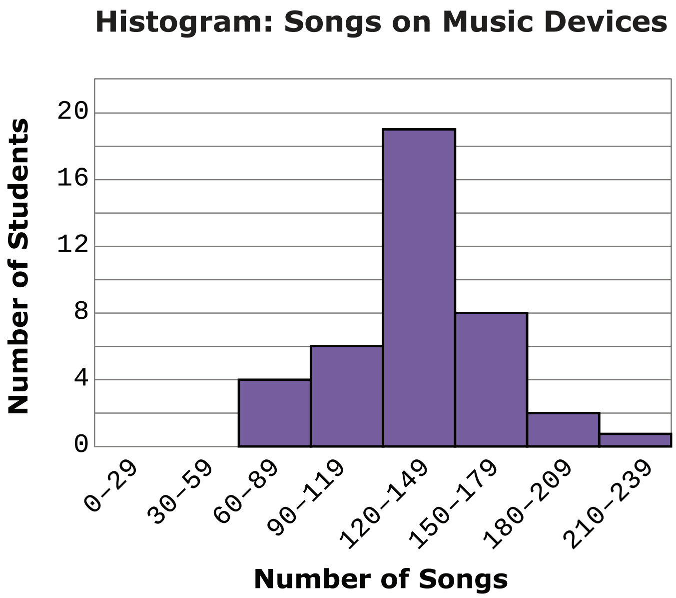
Photo Credit: TAJ Photoz / Shutterstock.com

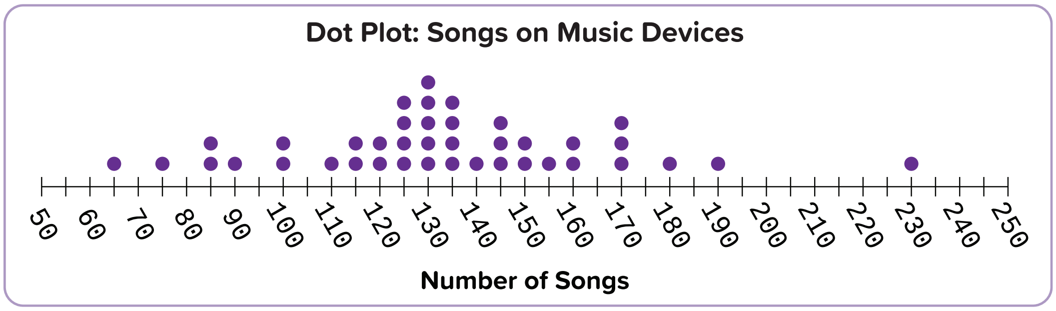
Let's Chat

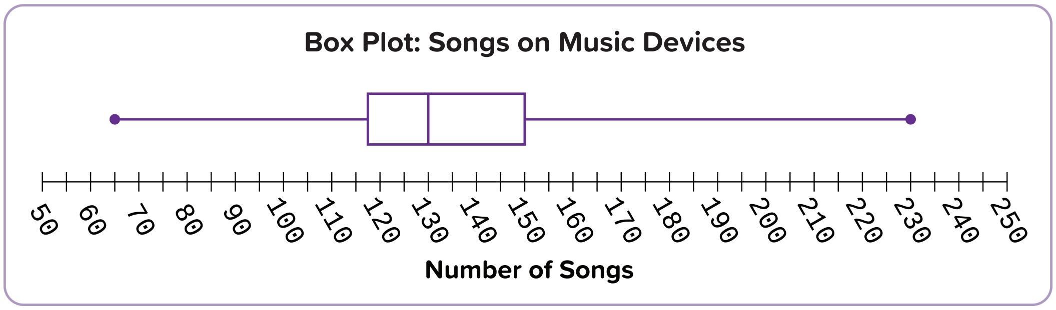
- How can you decide which is the best type to use to display different data sets?
- What would be important to know in order to decide which display is best to use?

BUILD AND CONNECT

Choosing Data Displays Suppose three students were collecting data on the statistical question, "How many songs do Primary 6 students have on their phones or music devices?" The three students graphed the data they collected using different data displays, as shown in the graphs.







Histogram Which of the following questions could be answered using the histogram? Identify all that apply.

- **A.** What is the most common interval for the number of songs?
- **B.** How many students are represented in the data?
- **C.** How many students had 180 songs or more on their music devices?
- **D.** How many students had exactly 120 songs on their music devices?
- **E.** What was the greatest number of songs a student had?
- **F.** How many students had from 90 to 179 songs?

Dot Plot Write two questions that can only be answered using the dot plot, but not the other two displays.

Box Plot Write two questions that can be answered using the box plot and one that cannot. Be sure to use new questions that you haven't seen or written yet.

Choose a Graph Label each question with the type of graph that would display the answer best.

dot plot | histogram | box plot

A. How many students have exactly 150 songs on their devices?

B. What is the median number of songs?

C. How many students had from 90 to 119 songs on their devices?

Let's Chat Discuss your answers with a partner. What are the benefits of using each type of graph? What are the limitations?

Analyze Data Values Suppose the students gathered information to find how long Primary 6 students spent listening to music each week. The data they collected is given in the table.

Number of Minutes Students Spent Listening to Music per Week									
120	15	45	30	60	90	0	125	30	240
75	45	80	10	20	35	45	90	100	115
75	40	70	100	120	120	150	15	0	20
5	120	45	80	10	45	50	100	15	0
20	35	120	150	30	60	90	20	35	40

Let's Chat If you wanted to make a data display of this data, what types of information would you need to know before you proceeded? Why might you choose one particular display over another? Discuss your thoughts with a partner.

Choose a Display Suppose you want to display this data to determine a typical number of minutes that students spent listening to music. Which graph would you use? Explain your reasoning.

Let's Chat With a partner, name the graph or graphs you would use to do the following:

- Show individual data values
- Show hundreds of observations
- Show clusters and gaps in the data

Check Your Understanding Follow your teacher's instructions to complete this activity.



Photo Credit: Tsuguliev / Shutterstock.com

UNIT

7

Theme 2 | Mathematical Operations and Algebraic Thinking: Statistics and Data Analysis

Measures of Central Tendency and Spread



Measures of Central Tendency and Spread

Unit Opener Video: Measures of Central Tendency and Spread

Quick Code
egm6121

Explains how useful interpretations and observations can be made by analyzing how data is distributed.

Key Vocabulary

absolute value, average, balance point, fair share, interquartile range (IQR), mean, mean absolute deviation (MAD), measures of center, measures of variability (spread), median, outlier, range, value, Spread

LESSON 1

Exploring the Balance of Data Sets

Quick Code
egm6123

Learning Target

- I can summarize the data in a data set using a single number.

Classroom Practices

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

ACCESS

Describe a Data Set Consider the ways you have collected and analyzed data in data displays. Suppose you want to summarize a data set using a number.

How could you assign a single **value** to represent a data set?

Describing a Data Set A father asked his son: "About how many students are in each of your classes at school?" The next day, the son counted the number of students in each class to get the following data set: 22, 19, 27, 25, 29, 21, and 25. Help the son report back to his father about the mean number of students in his classes by completing the questions.



Students in Class

A. What is the mean of students' number in a class?

B. What is the median of the data set?

C. What do you notice?

Let's Chat Discuss your answers with a partner.

- Did they get the same mean that you did?
- Did you use the same method to find the mean?

BUILD AND CONNECT

Balance Point One way to find the **mean** is to find a **balance point**.


Balance Point

Let's Chat Discuss with a partner. Would you have to shift the triangle to the left or right to balance the seesaw?

In this lesson, you are going to explore how to use balance points to numerically describe data sets. Complete the following activity to see how you can summarize a numerical data set by finding the value that "balances" the data in a dot plot.

Balance Point Use this Hands-On Activity to explore how you can summarize data. Use this online digital resource for Hands-On exploration.

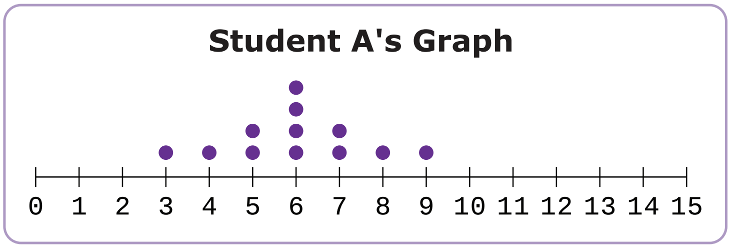
Let's Chat Discuss how you found the balance point of the data with a partner. How did moving the counters in the activity help you to find the center of the data?

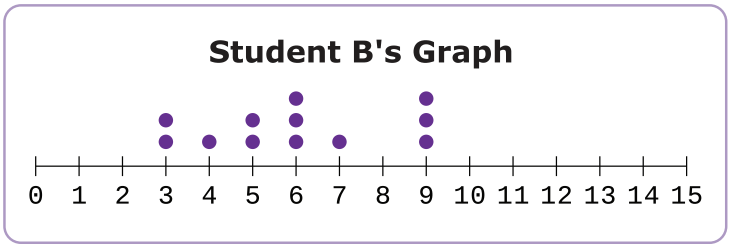
Interpret Balance Point Use what you learned in the activity to answer these questions.

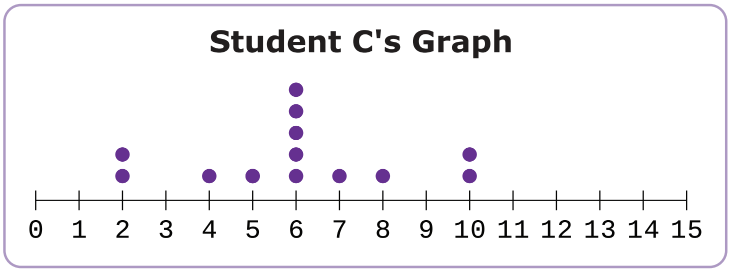
A. What does the balance point mean in terms of the data set in the activity?

B. How does the balance point summarize the values in a data set?

Analyze Graphs Suppose three students made these graphs based on some first names in their class. Use the graphs to complete the three tasks.







A. Explain how the graphs from Student A and Student C are similar to one another and describe how they differ from Student B's graph.

B. Describe the method that Student A and Student C could follow, using their graphs to find the balance point of the data.

C. Explain the method that Student B could follow, using his graph to find the balance point and describe how this is different from the method the other students could use.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 2

Interpreting Mean

Quick Code
egm6125

Learning Targets

- I can explore **mean** as a **fair share**.
- I can determine an algorithm for calculating the mean of a data set.

Classroom Practices

- Use appropriate tools strategically.
- Look for and make use of structure.

ACCESS

The Balance Point and Mean A measure of center of tendency summarizes a data set with a single value by telling you how data are clustered.



Football Players

Mean of Jersey Numbers

$$\frac{4 + 11 + 16 + 20}{4}$$

$$\text{Mean} = 12\frac{3}{4}$$

The mean is one measure of central tendency. Locating the balance point is one way to determine the mean of a data set.

Let's Chat

- Why do you think the balance point is a good mean value?
- Discuss your thoughts with a partner.

BUILD AND CONNECT

Part A: Mean as Balance Point Suppose that every year your class makes greeting cards for charity.

Find the Mean Last year, your teacher divided your class into 5 groups. Each group made the following numbers of cards: 32, 34, 36, 38, and 40. Calculate the mean number of cards that the student groups made.


Students Making Cards

Analyze Solutions Saeed said he found the mean of these numbers without creating a dot plot and moving counters. Explain how Saeed might have found the mean.

Analyze Another Situation How could Saeed find the mean of two data values, 39 and 51, without graphing? Explain a process he could use and how this value could be considered a mean.

Let's Chat Discuss with a partner why it is not always necessary to graph and move counters in order to find the mean of a data set. Will this method work for every data set?

Part B: Mean as Fair Share The balance point is only one way to think of the mean. Let's explore another.

Suppose your class is about to take a standardized test. Your teacher groups the students into 5 tables and asks each group of students to bring in #2 pencils for the test.

Use counters to help you complete the following exercises.



Sharpened Pencil

Finding the Number of Pencils Suppose each student in the first two groups of students brings in the number of pencils as described in this table.

Group 1		Group 2	
Student A	3	Student G	3
Student B	12	Student H	8
Student C	6	Student I	2
Student D	8	Student J	4
Student E	2	Student K	0
Student F	5	Student L	10

Use counters to represent the number of pencils brought in by each member. Then, complete the tasks defined for each group.

A. Group 1: Adjust the counters in your representation so that each group member gets the same number of pencils. How many pencils does each student get?

B. Group 2: Is it possible to adjust the counters in your representation so that each group member gets the same number of pencils, and all the pencils are used? Explain how this fair share is different from Group 1.

Solve a Problem Group 3 didn't keep track of how many pencils each of the 6 students brought, but they knew that they had a total of 42 pencils. How many pencils should each student get, if the pencils are distributed equally?

A. 4 pencils **B.** 6 pencils **C.** 7 pencils **D.** 42 pencils

Analyze Methods The table shows the number of pencils the students in the fourth group brought to class.

Group 4	
Student M	9
Student N	3
Student O	12
Student P	8
Student Q	8

When asked to find the mean of their group's pencils, Student M and Student P used different methods.

Student M's Strategy Student M collected all of the pencils and passed out each pencil one by one to each of the 5 students until there were none left.

Student P's Strategy Always looking for shortcuts, Student P didn't want to take the time to pass out each pencil one by one to his group members. He wanted to figure out how many each person would get and give them that many. To do so, he added up the total number of pencils in his group and divided that number by the number of students in his group.

Write your responses to the following questions.

A. Does Student M's method give each student in the group a fair share of the pencils? How about Student P's method? Explain.

B. What are the numerical expressions that model Student P's method?

Use an Expression Simplify the numerical expressions that model Student P's strategy to find the mean of the number of pencils Group 5 brought to class. Show your work and explain how you found your answer.

Group 5	
Student R	9
Student S	2
Student T	10
Student U	5
Student V	9

Let's Chat

- Discuss the expressions you wrote and simplified to find the mean with a partner.
- How does this value compare to what you would have gotten if you had found the balance point of the data?

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 3

Exploring Median, Mode, and Outliers

Quick Code
egm6127

Learning Target

- I can determine how **outliers** and shape of a graph can help to determine whether mean or **median** is a better measure of central tendency.

Classroom Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

ACCESS

Siblings You wonder how the number of siblings you have compares to other families, so you ask your classmates how many siblings they have. You record the data and start to calculate the mean. Most of the values are between zero and two, but one student has nine siblings!



Siblings

Photo Credit: stockers asia / Shutterstock.com

Let's Chat

- How would you calculate the mean of the data?
- Nine siblings is an outlier. What is an outlier?
- Do you think the data in your class would be similar?

BUILD AND CONNECT

Mean, Median, and Outliers Here is an example of mode, median, mean and outliers using a set of data.

0, 1, 7, 4, 18, 12, 0, 0, 9, 12, 11, 13, 17, 15

Order: 0, 0, 0, 1, 4, 7, 9, 11, 12, 12, 13, 15, 17, 18

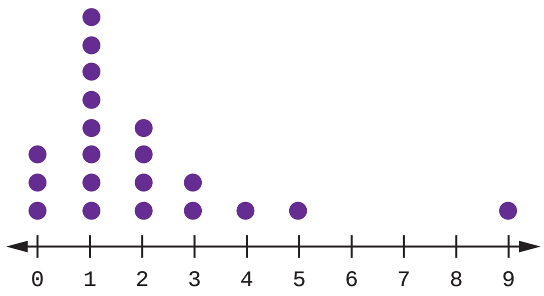
Mode = 0

Median = $\frac{9 + 11}{2} = 10$

Mean = $\frac{119}{14} = 8\frac{1}{2}$

Outliers = None

Number of Siblings of Primary 6 Students



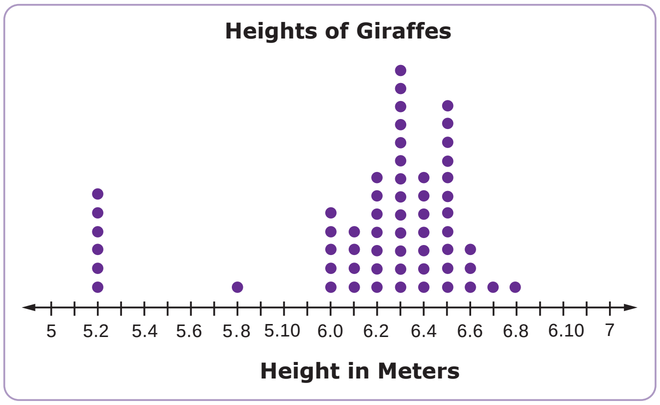
Siblings Here is the data from a Primary 6 class. In looking at the dot plot of the data about students' siblings, how does the outlier affect the mean?

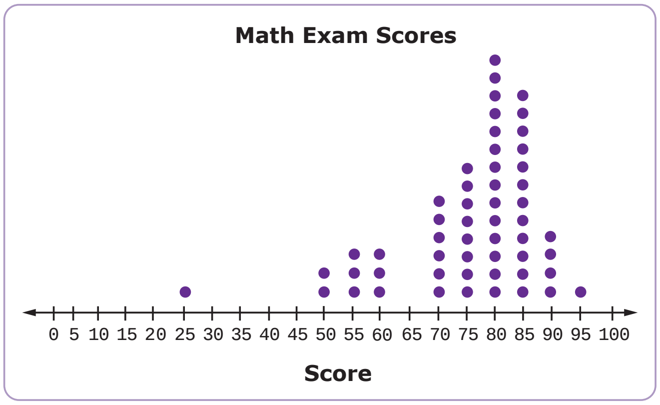
- **A.** The outlier would not have an effect on the mean.
- **B.** The outlier would cause the mean to be lower.
- **C.** The outlier would cause the mean to be much higher.
- **D.** The outlier would cause the mean to be higher, but the amount would be negligible.

Effect on Median Does the outlier change the median for this data set? Explain your reasoning.

Which Measure of Center? Which measure of central tendency do you think is better to use with the data that includes an outlier—the mean or the median? Explain your reasoning.

Consider the two dot plots.





Outlier's Effect In which set of data, the Heights of Giraffes or the Math Exam Scores, will the outlier make a greater difference on the mean? Explain your reasoning.

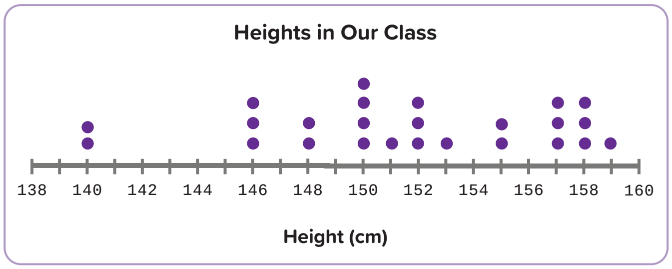
Why Are There Outliers? Think about the context of these graphs. Does it make sense that there would be outliers? Explain your reasoning.

Excluding the Outlier Asmaa found the mean of the test scores to be 75.9, including the outlier. The sum of the data was 4,100 and there are 54 pieces of data. She said it was easy to figure out what the mean would be if she excluded the outlier. Explain an easy way to find the mean without the outlier.

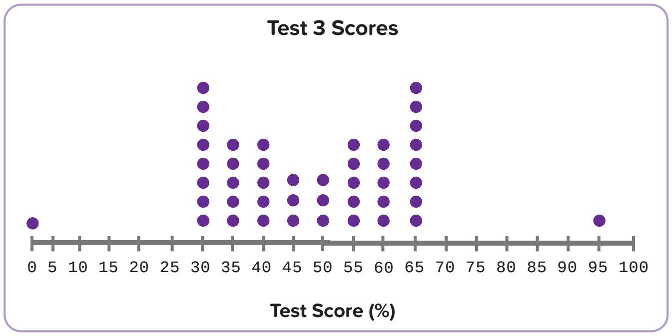
Now, think about how you can use the graph of data to determine whether the mean or median better describes a data set.

Changes How would these outliers affect the mean if they were included in the calculation? Use reasoning to assign each of the given descriptions with the related graph.

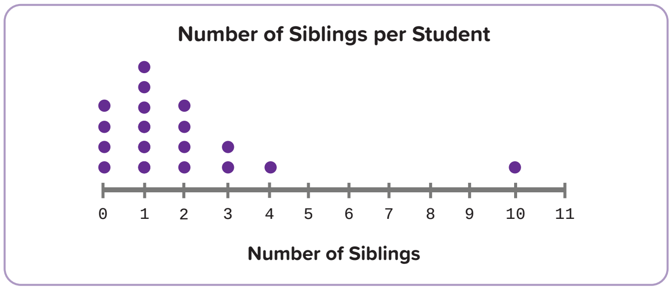
Mean increases	Mean decreases	Mean stays the same



(A) ______



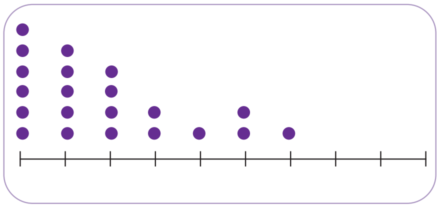
(B) ______



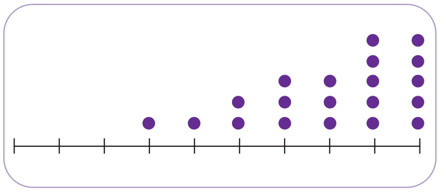
(C) ______

Mean or Median For each of these data displays, choose the measure of central tendency you think would be better to use.

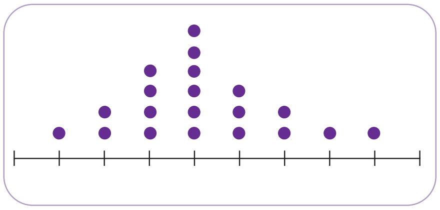
Mean   Median   Either



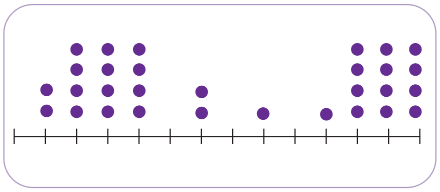
(A) ______



(B) ______



(C) ______



(D) ______

Let's Chat Discuss with a partner your responses to the previous tasks. Revise your responses as needed. Be prepared to share your response with the class.

Check Your Understanding Follow your teacher's instructions to complete this activity.

LESSON 4

Exploring The Range

Quick Code
egm6131

Learning Target

- I can define and compute the **range** of data sets as an introduction to the importance of **measures of variability**.

Classroom Practices

- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

ACCESS

Median Temperature in Spring and Summer You have previously explored how the mean and median can provide useful information by describing a data set with a single value. Do you think these measures of center give you a complete picture of a data set?



Thermometer

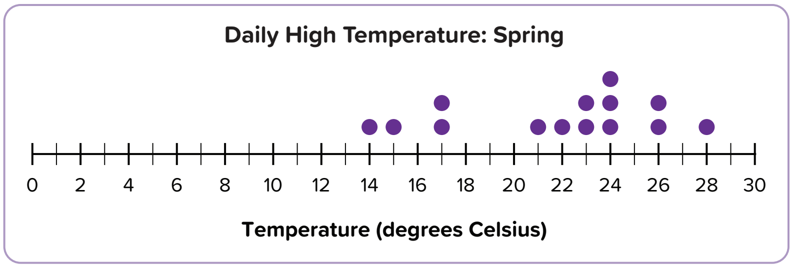
Photo credit: Marian Weyo / Shutterstock.com

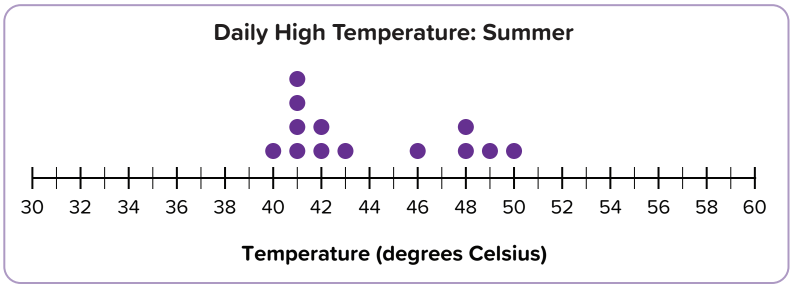
A group of students recorded the daily high temperatures for two weeks in the spring and two weeks in the summer. They found that the median of the daily high temperatures for each of the two-week periods were the same: 20 °C.

Let's Chat Can you conclude that the weather was the same for each of the two weeks in the spring and summer?

BUILD AND CONNECT

Explore Range The students made dot plots of the data to help them compare the temperatures they recorded in the spring and the summer. What do you notice about these dot plots?





Range There is a value called range that will help you understand the spread of the data. The range for the spring data is 14. The range for the summer data is 10.

Understand Range Use the data from the dot plots to answer these questions.

A. How do you think the range was calculated for spring temperatures?

B. How do you think the range was calculated for summer temperatures?

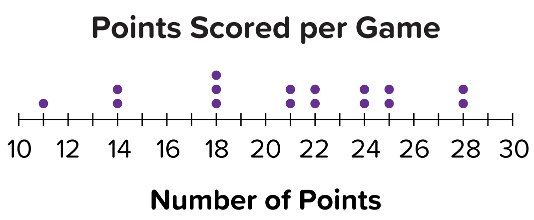
Analyze Range Explain what the range tells you about a data set.

Let's Chat Discuss with a partner your responses.

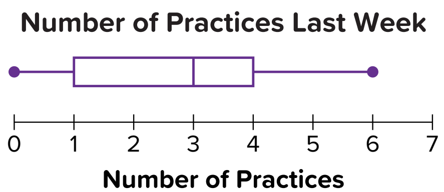
- Why is range a good measure to represent the data for the spring and summer temperatures?
- What do you think the higher range means in this scenario?

Analyze Other Situations Now that you know what range is, you can try finding it yourself.

Points Scored Omar used a dot plot to show the total points he scored per basketball game this season. Omar told his teacher that the range is 20. His friend Rami said that the range is 7. The teacher told them that they both calculated the range incorrectly. Explain the errors each student made. What is the correct range?



Practices Ali drew a box plot to show the number of times that students in a band practiced their musical instruments last week. What is the range of the number of times students practiced?



Quiz Scores The table shows Nour's quiz scores. What is the range of these quiz scores?

Quiz Number	Score
1	18
2	15
3	17
4	20
5	18
6	19
7	18
8	16

Analyze Displays Use what you know about range and the different data displays used in the previous exercises to answer these questions.

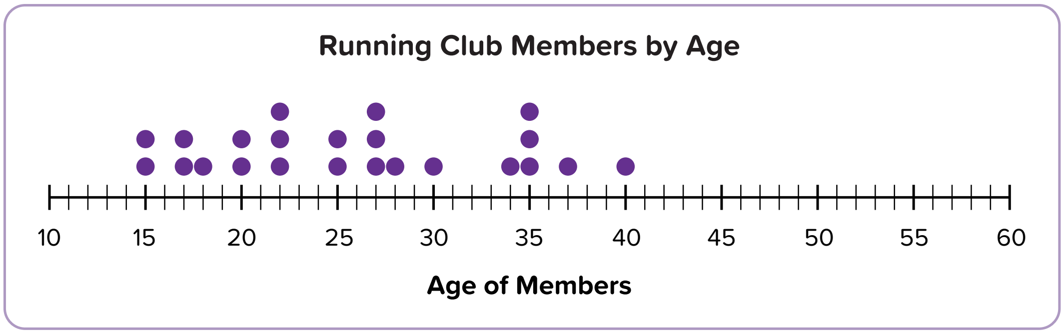
A. Which representation of the data was the easiest for you to use to find range? Which was the hardest?

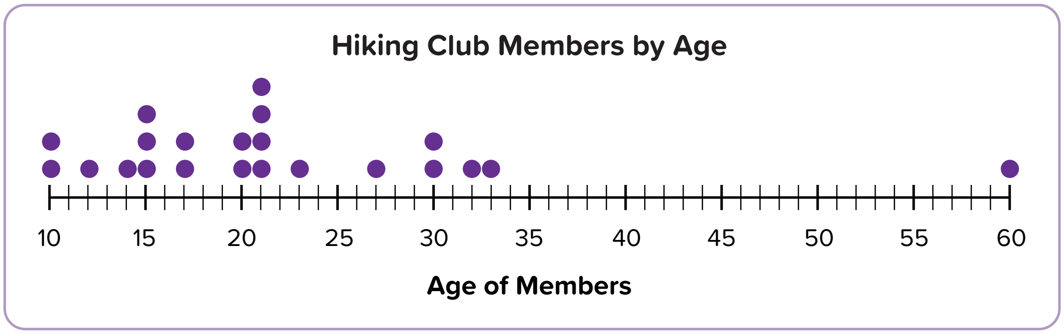
B. Is it possible to use a histogram to find range? Why or why not?

Understanding the Range Now that you know more about range, you can further analyze how range changes the view of the data.

Let's Chat Can you think of a situation when the range might give you a distorted view of the data? Discuss your ideas with a partner.

Comparing Ranges Consider the dot plots featuring the ages of members in the Running Club versus the Hiking Club.





For which dot plot does the range give a more accurate picture of the ages of the majority of the people in each of the clubs? Explain your reasoning.

Check Your Understanding Follow your teacher's instructions to complete this activity.

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